Πgora: An Integration System for Probabilistic Data

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Probabilistic Data Formalisms

Real-world applications model probabilistic data using a plethora of different formalisms, e.g.,

- **Bayesian networks** are a natural fit for managing expert knowledge, where the probabilistic relationship between input random variables, which are observable quantities, unknown parameters, or hypotheses, exhibits conditional independence.
  - Examples from the **UCI machine learning repository** at `http://archive.ics.uci.edu/ml/datasets.html`
- The **pc-tables** are relations extended with a special column that encodes the uncertainty of the records using probabilistic events.
  - **NELL tables** at `http://rtw.ml.cmu.edu/rtw/` consist of records extracted from hundreds of millions of web pages.
  - **Google Squared tables** aggregate unstructured, possibly contradictory information representing answers to keyword search queries.
- **Finite State Transducers (FSTs)** are stochastic automata used by optical character recognition programs, such as those powering **Google Books**, to capture probability distributions over all possible strings that could be represented in a given image.
  - Examples at `http://hazy.cs.wisc.edu/hazy/staccato/`

They support probabilistic processing to varying degrees:

- The **pc-tables formalism supports select-project-join queries** whose answers can be represented as pc-tables.
  - as implemented by the MayBMS/SPROUT query engine
- **Bayesian networks support inference queries** that ask for the conditional probability of an event given another event.
  - as implemented by the SMILE Bayesian inference engine
- **FSTs support selection queries** that ask for the probability that a certain string occurs in their possible runs.
  - as implemented by the Staccato system

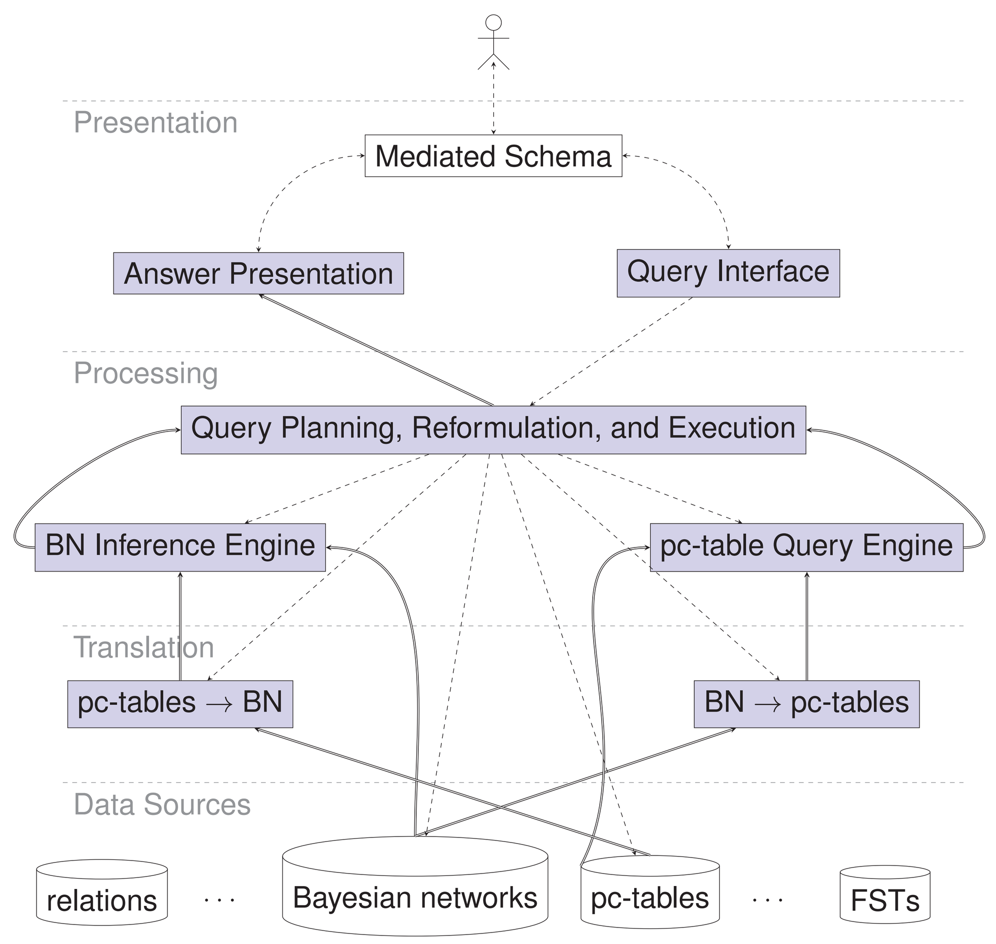
They admit a common interpretation via the possible worlds semantics:

- pc-tables represent finite probability distributions over sets of possible tables.
- Bayesian networks represent finite probability distributions over sets of correlated observations.
- FSTs represent finite probability distributions over sets of possible strings represented in an image.

Πgora's Capabilities

Queryable uniform interface to heterogeneous probabilistic data

- provides uniform interface = **mediated relational schema**
  - Each local source is registered to the system with a relational schema that becomes part of the mediated schema.
  - pc-tables export a relational schema without the events column.
  - Bayesian networks export a relational schema consisting of one attribute per node.
  - FSTs export schemas with one attribute.
- integrates data available in different probabilistic formalisms
- enables expressive querying across them
  - select-project-join queries
  - exact/approximate probability computation aggregate
  - a new `GIVEN` clause that allows to formulate conditionals.
- provides a query evaluation mechanism over the mediated schema
  - **Strategy 1**: Use different engines to evaluate subqueries natively supported by the formalisms of the input data sources. Translate intermediate results to pc-tables and complete the evaluation.
  - **Strategy 2**: Offline translation of all input data sources into either the pc-tables or Bayesian networks, followed by evaluation using either a query or an inference engine.
- provides transformations of sources to existing formalisms followed by evaluation using a single query/inference engine.
  - pc-tables, Bayesian networks, and FSTs are complete representation systems but of incomparable succinctness.
  - exponential-time translations between the formalisms
  - polynomial-time translations of Bayesian networks and FSTs into *pc-tables with event definitions*



Legend: Double/dashed arrows denote data/control flow.

Demonstration Scenario: Medical Data

Query: probability of a pregnant woman suffering from a left breast tumour, given that she also suffers from hypothyroidism.

```
SELECT conf() FROM Hypothyroid H, Breast_cancer B
WHERE B.tumour='true' AND B.breast='left' AND H.tumour='true' AND H.pregnant='true'
GIVEN B.age=H.age AND H.hypothyroid='primary'
```

Data sources: Bayesian networks `Hypothyroid` and `Breast_cancer`.

Strategy 1: purely Bayesian evaluation.

- Phrase the SQL query as a sum of inference queries:

$$\sum_{\text{B.age}} (P(\text{B.tumor} = \text{true} \wedge \text{B.breast} = \text{left} \wedge \text{H.tumor} = \text{true} \wedge \text{H.pregnant} = \text{true} \mid (\text{B.age} = \text{H.age} \wedge \text{H.hypothyroid} = \text{primary})))$$

- For a given value x for `age`, we have the inference query:

$$P(\text{B.tumor} = \text{true} \wedge \text{B.breast} = \text{left} \wedge \text{H.tumor} = \text{true} \wedge \text{H.pregnant} = \text{true} \mid (\text{B.age} = x \wedge \text{H.age} = x \wedge \text{H.hypothyroid} = \text{primary}))$$

- Since the two Bayesian networks are independent, we can regroup as follows:

$$P(\text{B.tumor} = \text{true} \wedge \text{B.breast} = \text{left} \mid \text{B.age} = x) * P(\text{H.tumor} = \text{true} \wedge \text{H.pregnant} = \text{true} \mid (\text{H.age} = x \wedge \text{H.hypothyroid} = \text{primary}))$$

Strategy 2: evaluation using pc-tables translations of Bayesian networks.

- Resolve the `GIVEN` clause using the conditional probability formula:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)}$$

Strategy 3: hybrid evaluation assuming `Breast_cancer` is a pc-table.

- Split the query into the subqueries over each of `Hypothyroid` and `Breast_cancer`.
- For each value of x for `age` we have the inference query over `Hypothyroid`:

$$\forall x: P_H(x) = P(\text{H.tumor} = \text{true} \wedge \text{H.pregnant} = \text{true} \mid \text{H.age} = x \wedge \text{H.hypothyroid} = \text{primary})$$

- Rewrite the subquery over `Breast_cancer` by resolving the `GIVEN` clause:

```
CREATE TABLE T1 AS SELECT B.age, conf() AS p1 FROM Breast_cancer B
WHERE B.tumor='true' AND B.breast='left' GROUP BY B.age

CREATE TABLE T2 AS SELECT B.age, conf() as p2 FROM Breast_cancer B GROUP BY B.age

CREATE TABLE T3 AS SELECT T1.age, p1/p2 AS PB FROM T1, T2 WHERE T1.age = T2.age
```

- The query answer is obtained by joining the independent intermediate results:

$$\sum_{age} P_B(age) * P_H(age)$$

where $P_B(age)$ denotes P_B for the tuple (age, P_B) in T_3.