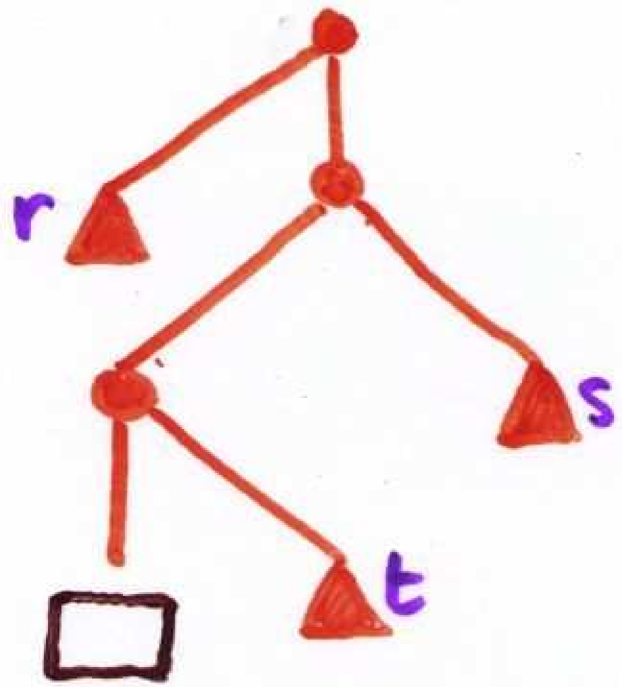
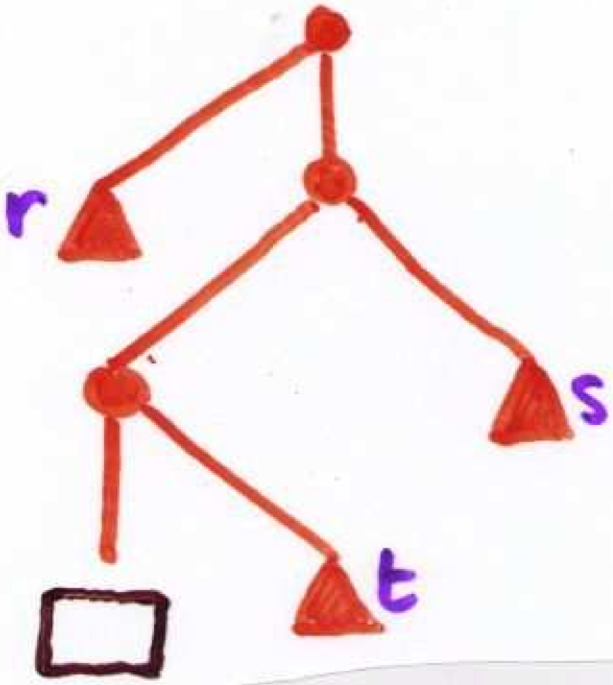
Calculate the Zipper type

```
data Tree = Leaf
          | Node Tree Tree
```



Calculate the Zipper type

```
data Tree = Leaf
          | Node Tree Tree
```



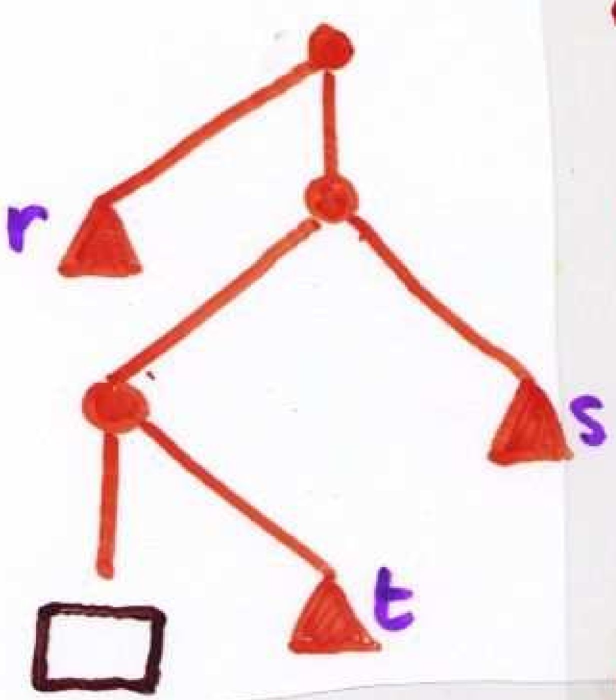

```
data Step = Left Tree

          | Right Tree

type Zipper = [Step]
```

Calculate the Zipper type

```
data Tree = Leaf
          | Node Tree Tree
```



[Left t, Left s, Right r]

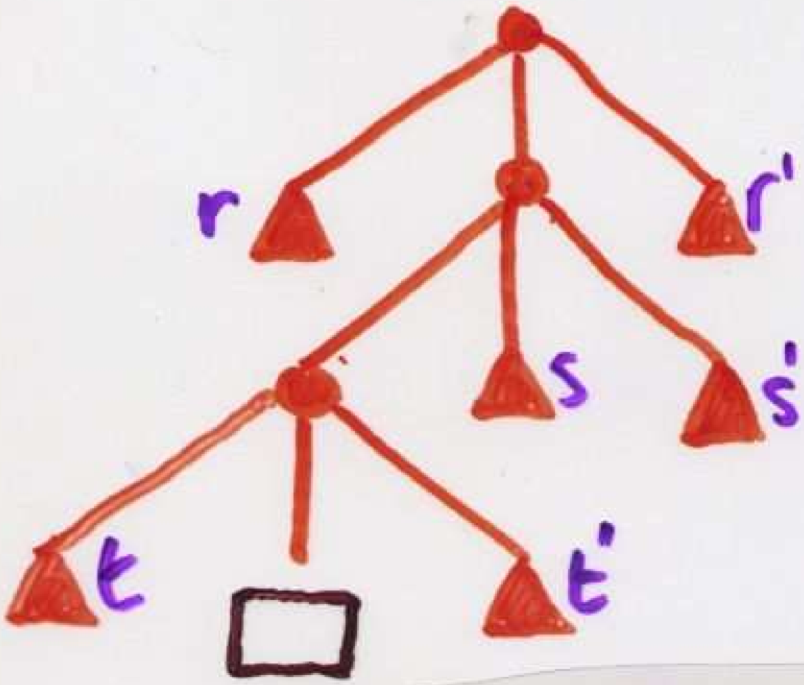
```
data Step = Left Tree

          | Right Tree
```

```
type Zipper = [Step]
```

Calculate the Zipper type

```
data Tree = Leaf
          | Node Tree Tree Tree
```




```
[Middle t t', Left s s', Middle r r']
```

```
data Step = Left Tree Tree
          | Middle Tree Tree
          | Right Tree Tree

type Zipper = [Step]
```

Partial Derivative wrt free variable

$$
\begin{aligned}
\partial_x\, y &= \delta_{xy} \\
\partial_x\, 0 &= 0 \\
\partial_x\, (S + T) &= \partial_x S + \partial_x T \\
\partial_x\, 1 &= 0 \\
\partial_x\, (S \times T) &= \partial_x S \times T + S \times \partial_x T \\
\partial_x\, (\{y = S\}\, T) &= (\{y = S\}\, \partial_x T) + \\
&\quad\; (\{y = S\}\, \partial_y T) \times \partial_x S \\
\partial_x\, (\mu y.\, T) &= [\{y = \mu y.\, T\}\, \partial_y T] \times \\
&\quad\; (\{y = \mu y.\, T\}\, \partial_x T)
\end{aligned}
$$

Plugging In

$$(\langle T \circ\!\!- x]) :: \partial_x T \to x \to T$$

$$\begin{aligned}
() \quad \langle x \circ\!\!- x]\; x &= x \\
\mathsf{inl}\; s' \; \langle S + T \circ\!\!- x]\; x &= \mathsf{inl}\,(s' \langle S \circ\!\!- x]\; x) \\
\mathsf{inr}\; t' \; \langle S + T \circ\!\!- x]\; x &= \mathsf{inr}\,(t' \langle T \circ\!\!- x]\; x) \\
\mathsf{inl}\,(s', t) \langle S \times T \circ\!\!- x]\; x &= (s' \langle S \circ\!\!- x]\; x, t) \\
\mathsf{inr}\,(s, t') \; \langle S \times T \circ\!\!- x]\; x &= (s, t' \langle T \circ\!\!- x]\; x) \\
\mathsf{inl}\; t' \; \langle \{y = S\}\, T \circ\!\!- x]\; x &= t' \langle T \circ\!\!- x]\; x \\
\mathsf{inr}\,(t', s') \; \langle \{y = S\}\, T \circ\!\!- x]\; x &= t' \langle T \circ\!\!- y]\; s' \langle S \circ\!\!- x]\; x \\
(t's, t') \; \langle \mu y.\, T \circ\!\!- x]\; x &= ts \,\langle y.\, T]\, (\mathsf{in} \\
&\qquad (t \langle T \circ\!\!- x]\; x))
\end{aligned}$$

$$(\langle y.\, T]) :: [\{y = \mu y.\, T\}\, \partial_y T] \to \mu y.\, T \to \mu y.\, T$$

$$\begin{aligned}
[\,] \; \langle y.\, T]\; t &= t \\
(t' : t's) \; \langle y.\, T]\; t &= t's \; \langle y.\, T]\; \mathsf{in}\,(t' \langle T \circ\!\!- y]\; t)
\end{aligned}$$