Dynamics, robustness and fragility of trust

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Outline

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Adverse selection

	TRUSTE-certified	uncertified
honest	94.6%	97.5%
malicious	5.4%	2.5 %

Table: Trustworthyness of TRUSTE [Edelman 2007]

Adverse selection

Google		
	sponsored	organic
top	4.44%	2.73%
top 3	5.33%	2.93 %
top 10	5.89%	2.74 %
top 50	5.93%	3.04 %

Table: Malicious search engine placements [Edelman 2007]

Adverse selection

Yahoo!		
	sponsored	organic
top	6.35%	0.00%
top 3	5.72%	0.35 %
top 10	5.14%	1.47 %
top 50	5.40%	1.55 %

Table: Malicious search engine placements [Edelman 2007]

Adverse selection

Ask		
	sponsored	organic
top	7.99%	3.23%
top 3	7.99%	3.24 %
top 10	8.31%	2.94 %
top 50	8.20%	3.12 %

Table: Malicious search engine placements [Edelman 2007]

Adverse selection

"Pillars of the society"

Social hubs are are often corrupt.

Questions

- Why does adverse selection happen?
- Can it be eliminated? Limited?
- Can we hedge against it?

Outline

Introduction

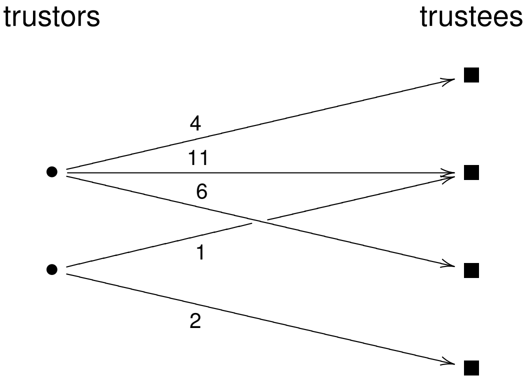
Private trust process

- Trust dynamics
- Trust distribution
- Interpretation

Public trust process

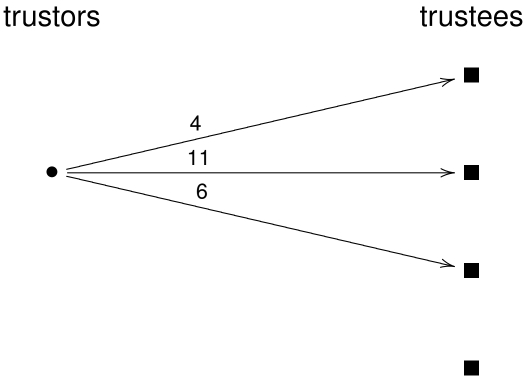
Conclusions

Trust (rating) vectors



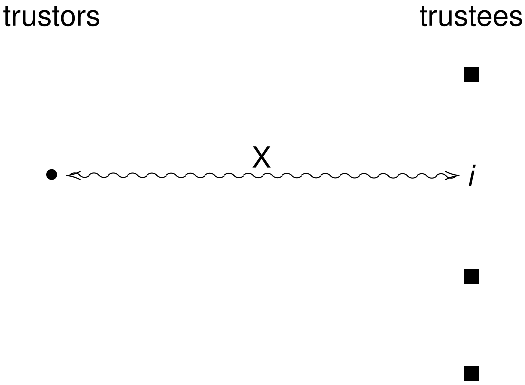
τ^1	4	11	6	0
τ^2	0	1	0	2

Private trust dynamics



$\tau(t)$	4	11	6	0

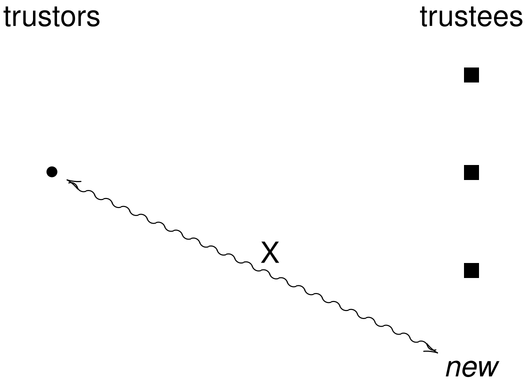
Private trust dynamics



$$\mathrm{Prob}\Big(X(t+1)=i\Big)=C(t)\tau_i(t)$$

(where $C(t)=\frac{1-\alpha}{\sum_{i\in J}\tau_i(t)}$)

Private trust dynamics



$$\mathrm{Prob}\Big(X(t+1) = new\Big) \quad = \quad \alpha$$

Private trust dynamics

Trust updating process

$$\tau_i(t+1) \quad = \quad \begin{cases} \tau_i(t) & \text{if } i \neq X(t+1) \\ 0 & \text{if } i = X\text{, not satisfactory} \\ 1 & \text{if } i = X\text{, satisfactory, new} \\ 1 + \tau_i(t) & \text{if } i = X\text{, satisfactory, not new} \end{cases}$$

Trust distribution

Task

Estimate

$$w_\ell(t) \quad = \quad \#\{i \in \mathsf{J} \mid \tau_i(t) = \ell\}$$

Trust distribution

$$
\begin{aligned}
w_1(t+1) - w_1(t) &= J \cdot \mathrm{Prob}\big(X(t+1) = i \mid i \text{ new}\big) \cdot \gamma_\perp \\
&\quad - w_1(t) \cdot \mathrm{Prob}\big(X(t+1) = i \mid \tau_i(t) = 1\big) \\
&= J\alpha\gamma_\perp - w_1(t)C(t)
\end{aligned}
$$

Trust distribution

$$
\begin{aligned}
w_\ell(t+1) - w_\ell(t) &= w_{\ell-1}(t) \cdot \mathrm{Prob}\Big(X(t+1) = i \mid \tau_i(t) = \ell - 1\Big) \cdot \gamma_{\ell-1} \\
&\quad - w_\ell(t) \cdot \mathrm{Prob}\Big(X(t+1) = i \mid \tau_i(t) = \ell\Big) \\
&= w_{\ell-1}(t)C(t)(\ell-1)\gamma_{\ell-1} - w_\ell(t)C(t)\ell
\end{aligned}
$$

Trust distribution

The system

$$
\begin{aligned}
\Delta_t w_1(t) &= J\alpha\gamma_\perp - C(t)w_1(t) \\
\Delta_t w_\ell(t) &= w_{\ell-1}(t)C(t)(\ell-1)\gamma_{\ell-1} - w_\ell(t)C(t)\ell
\end{aligned}
$$

Trust distribution

. . . divided by J becomes

$$\begin{aligned}
\Delta_t v_1(t) &= \alpha\gamma_\perp - C(t)v_1(t) \\
\Delta_t v_\ell(t) &= v_{\ell-1}(t)C(t)(\ell-1)\gamma_{\ell-1} - v_\ell(t)C(t)\ell
\end{aligned}$$

where $v_\ell(t) = \frac{w_\ell(t)}{J} = \mathrm{Prob}(i \in \mathrm{J} \mid \tau_i(t) = \ell)$
form a stochastic process $v : \mathbb{N} \longrightarrow \mathcal{D}\mathrm{R}$

Trust distribution

…and since $v : \mathbb{N} \longrightarrow \mathcal{D}\mathrm{R}$ is a martingale, it extends to $v : \mathbb{R} \longrightarrow \mathcal{D}\mathrm{R}$ and the system becomes

$$\begin{aligned} \frac{dv_1}{dt} &= \alpha\gamma_\perp - \frac{c}{t} v_1 \\ \frac{dv_\ell}{dt} &= \frac{\gamma_{\ell-1} c(\ell - 1) v_{\ell-1} - c\ell v_\ell}{t} \end{aligned}$$

where $C(t) \approx \frac{c}{t}$, for $c = \frac{1-\alpha}{1+\alpha\gamma_\perp}$ (see Appendix)

Trust distribution

The steady state of $v : \mathbb{R} \longrightarrow \mathcal{D}R$ will be in the form $v_\ell(t) = t \cdot \upsilon_\ell$, where

$$
\begin{aligned}
\upsilon_1 &= \alpha\gamma_\perp - c\upsilon_1 \\
\upsilon_\ell &= \gamma_{\ell-1} c(\ell - 1)\upsilon_{\ell-1} - c\ell\upsilon_\ell
\end{aligned}
$$

Trust distribution

The steady state of $v : \mathbb{R} \longrightarrow \mathcal{D}R$ will be in the form $v_\ell(t) = t \cdot \upsilon_\ell$, where

$$\upsilon_1 = \frac{\alpha\gamma_\perp}{c+1}$$

$$\upsilon_\ell = \frac{(\ell-1)\gamma_{\ell-1} c}{\ell c + 1}\, \upsilon_{\ell-1}$$

Trust distribution

...which expands into

$$\upsilon_2 = \frac{\alpha\gamma_\perp}{c+1} \cdot \frac{\gamma_1 c}{2c+1}$$

$$\upsilon_3 = \frac{\alpha\gamma_\perp}{c+1} \cdot \frac{\gamma_1 c}{2c+1} \cdot \frac{2\gamma_2 c}{3c+1}$$

$$\vdots$$

$$\begin{aligned}
\upsilon_n &= \alpha\gamma_\perp \left(\prod_{\ell=1}^{n-1} \gamma_\ell\right) c^{n-1} \cdot \frac{(n-1)!}{\prod_{k=1}^{n}(kc+1)} \\
&= \frac{\alpha\gamma_\perp G_{n-1}}{c} \cdot \frac{(n-1)!}{\prod_{k=1}^{n}\left(k+\frac{1}{c}\right)} \\
&= \frac{\alpha\gamma_\perp G_{n-1}}{c} \cdot \frac{\Gamma(n)\Gamma\left(1+\frac{1}{c}\right)}{\Gamma\left(n+1+\frac{1}{c}\right)} \\
&= \frac{\alpha\gamma_\perp G_{n-1}}{c} \cdot B\left(n, 1+\frac{1}{c}\right)
\end{aligned}$$

Trust distribution

The solution

$$\begin{aligned} \upsilon_1 &= \frac{\alpha\gamma_\perp}{c+1} \\ \upsilon_n &= \frac{\alpha\gamma_\perp G_{n-1}}{c} B\left(n, 1+\frac{1}{c}\right) \\ &\xrightarrow{n\to\infty} \frac{\alpha\gamma_\perp G}{c} n^{-\left(1+\frac{1}{c}\right)} \end{aligned}$$

where

$$G = \prod_{\ell=1}^{\infty} \gamma_\ell > 0 \text{ follows from}$$

$$\frac{1}{e^{s_\ell}} \leq \gamma_\ell \leq 1 \text{ for some}$$

$$\sum_{\ell=1}^{\infty} s_\ell < \infty$$

Trust distribution

Theorem

The described process of trust building leads, in the long run, to the power law distribution of the number of trustees with the trust rating n

$$w_n \approx \frac{\alpha\gamma_\perp GJ}{c}\, n^{-\left(1+\frac{1}{c}\right)}$$

Trust distribution

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provided that the incidence of dishonest principals who act honestly long enough to accumulate a high trust rating — is low enough

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provided that the incidence of dishonest principals who act honestly long enough to accumulate a high trust rating — is low enough (so that $\gamma_\ell \stackrel{\ell\to\infty}{\longrightarrow} 1$ fast enough)

What does this mean?

Some things have a fixed scale

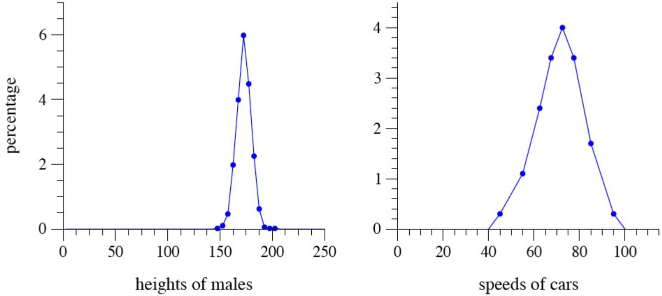

Figure: Normal distribution $f(x) = ae^{-bx^2}$

What does this mean?

Many social phenomena are scale-free

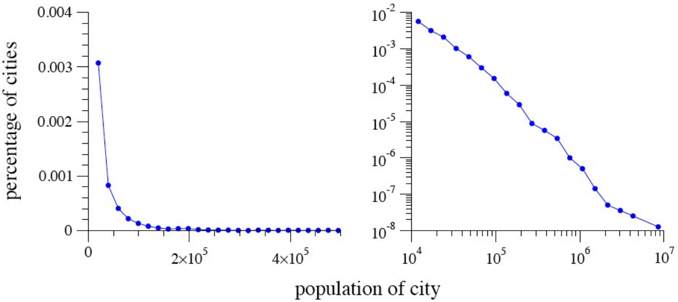

Figure: Power law $w(x) = ax^{-(1+b)}$

What does this mean?

Origin of scale-free distributions

V. Pareto: "The rich get richer"

What does this mean?

Origin of scale-free distributions

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Robustness of scale free distributions

The market is stabilized by the hubs of wealth.

What does this mean?

Origin of scale-free distributions

V. Pareto: "The rich get richer"

Robustness of scale free distributions

The market is stabilized by the hubs of wealth.

Fragility of scale free distributions

Theft is easier when there are very rich people.

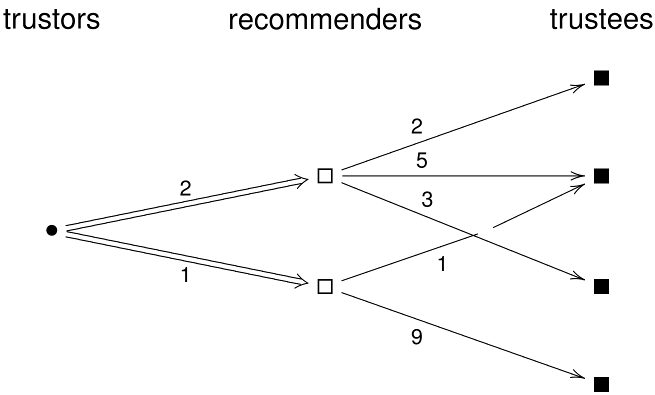
What does this mean?

But why is the distribution of a private trust vector a social phenomenon?

Outline

Public trust process

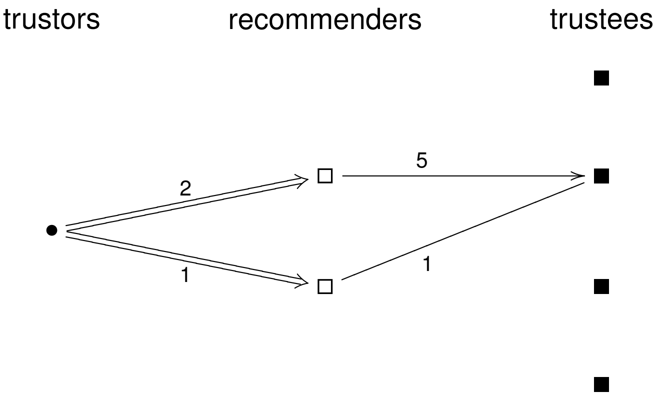
Using recommenders



2	A_1	2	5	3	0
1	A_2	0	1	0	9
σ	τ	4	11	6	9

Public trust process

Using recommenders



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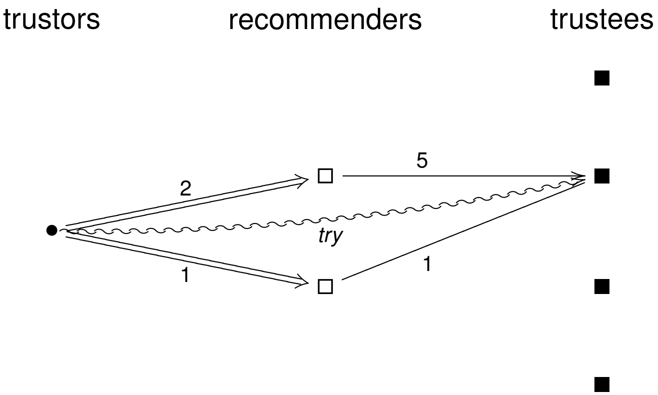
Public trust process

Using recommenders



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Public trust process

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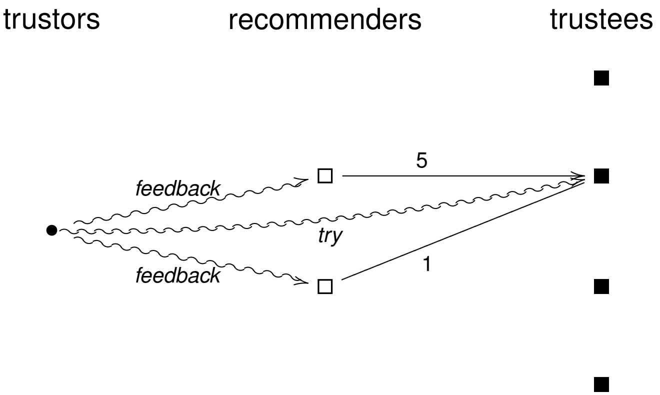

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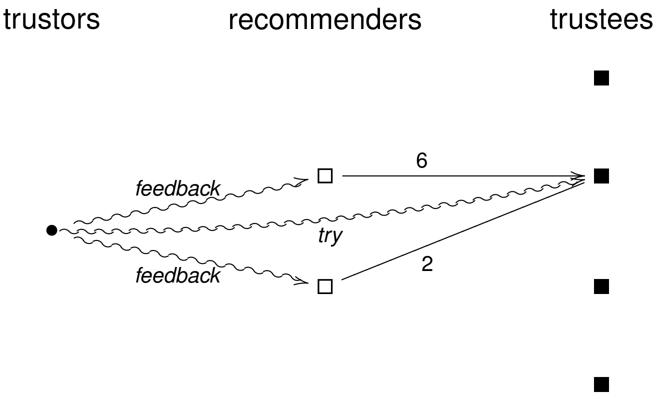
Public trust process

Using recommenders



2	A_1	2	6	3	0
1	A_2	0	2	0	9
σ	τ	4	14	6	9

Public trust distribution

Upshot

Recommenders' public trust vectors also obey the power law distribution.

Recommenders' reputation obeys the power law distribution.

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Recommenders' reputation obeys the power law distribution.

Consequence

Adverse selection

Outline

Conclusions

- Trust decisions should not be derived from public trust recommendations alone. They should be based on private trust vectors, that the user should maintain herself.

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- Trust decisions should not be derived from public trust recommendations alone. They should be based on private trust vectors, that the user should maintain herself.
- Public trust recommendations should be used to supplement and refine private trust.

Tasks

- mine the tightly knit subnets of trust networks:
  - uncover the *cliques of trust*
- diversify and localize value and trust
  - modern markets function without universal value — or abstract trust
- bridge the gap between public and private trust