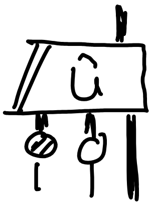
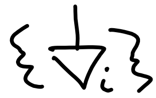
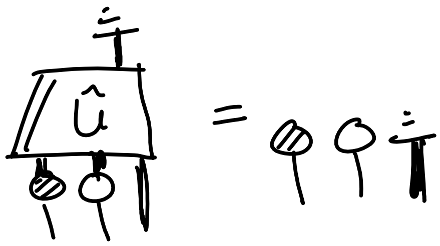
Quantum Processes & Computation

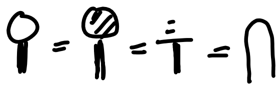
Model solutions, sheet 6

Oxford MT 2022

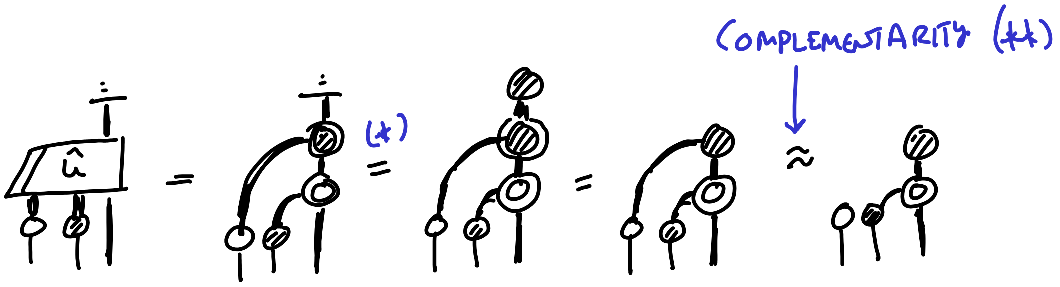
Ex 6.1

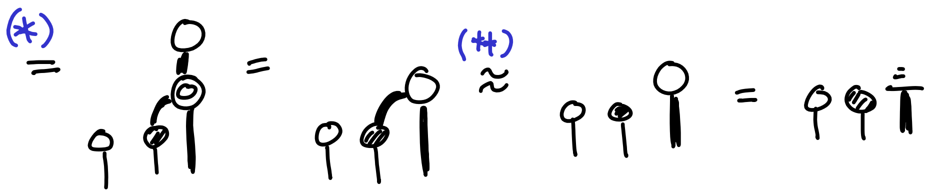
1. For  , the 1^{st} input wire is classical in  basis, 2^{nd} is classical in  basis, so causality means:



For bastard spiders with 1 quantum leg:  (*)

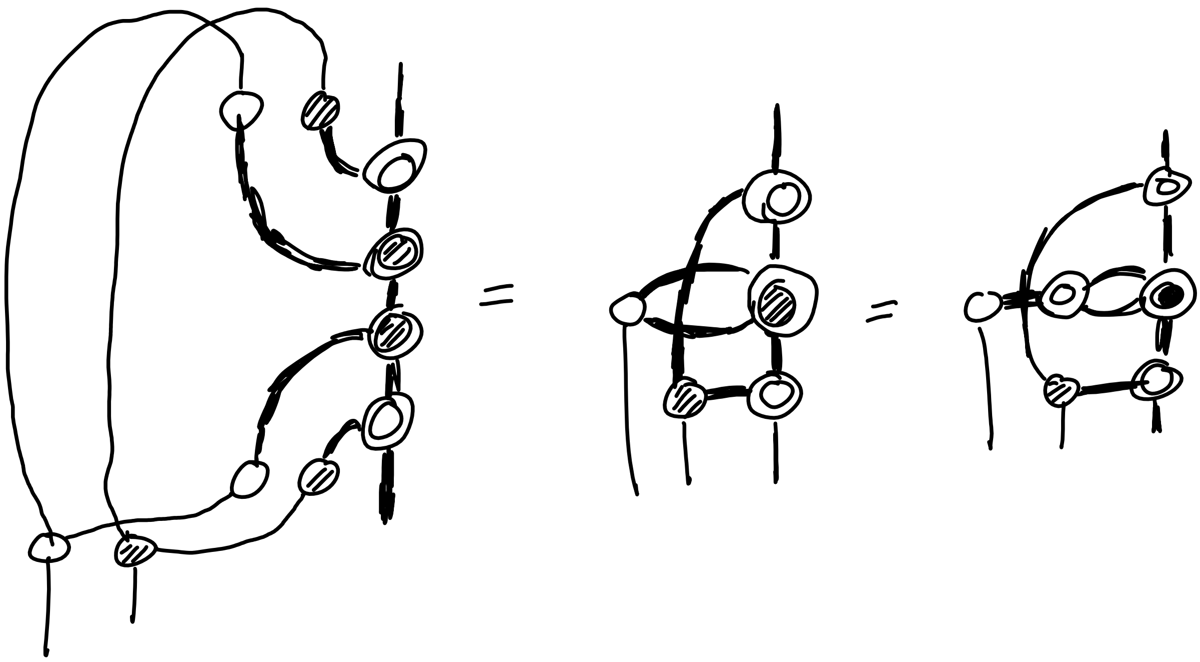
So:

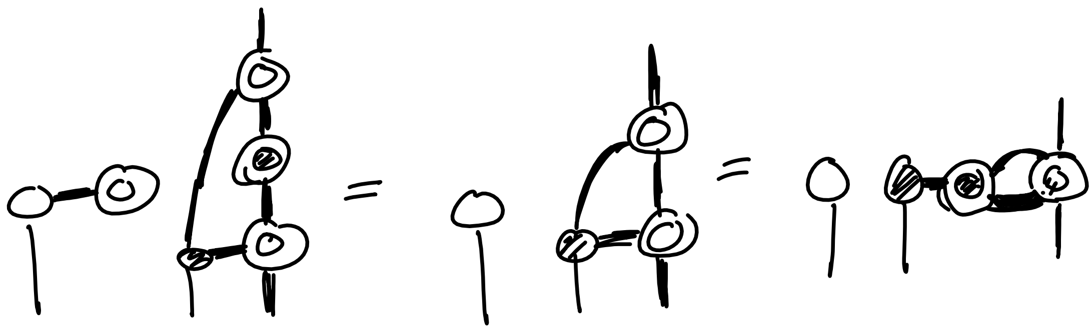


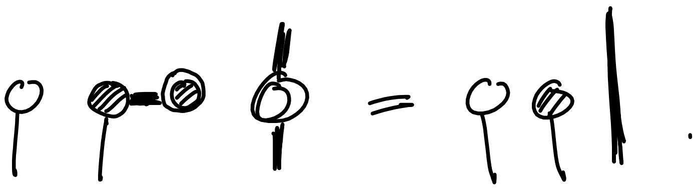


<u>6.1</u> (cont'd)

2.

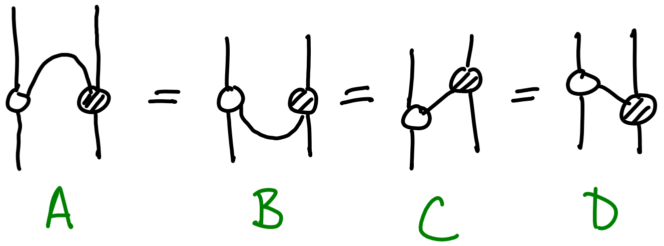


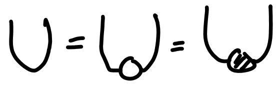
≈ 

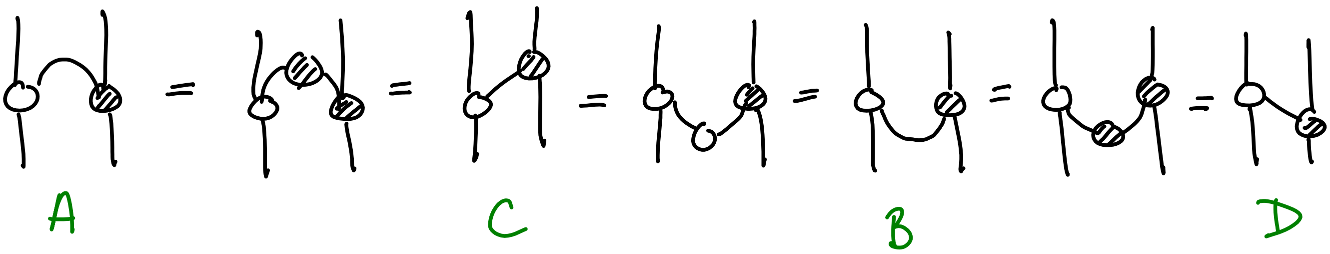
≈ 

<u>6.2</u>

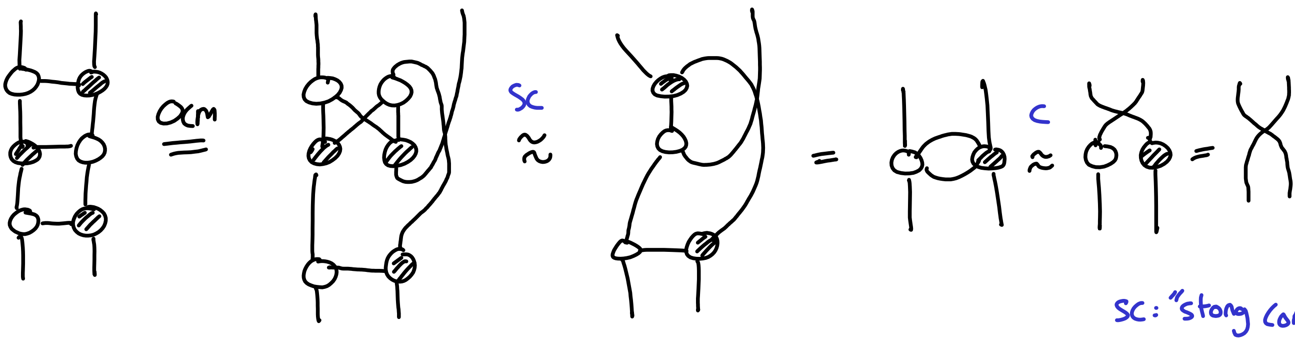
(i) Need to show:



Starting with A, and using spider fusion, , and , we have:

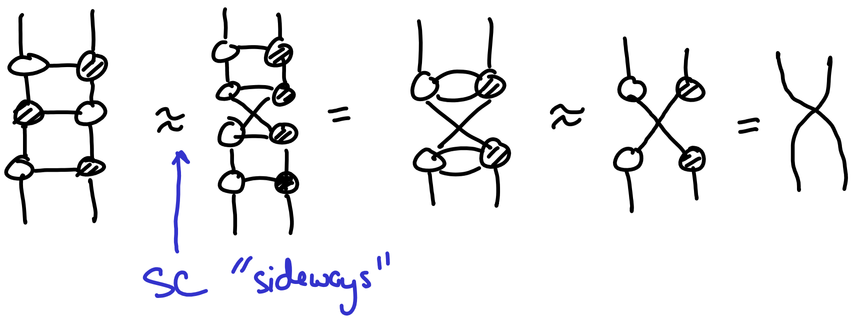


(ii)

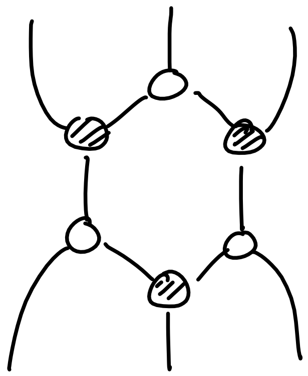


sc: "strong compl."
c: "compl."
(implied by sc)

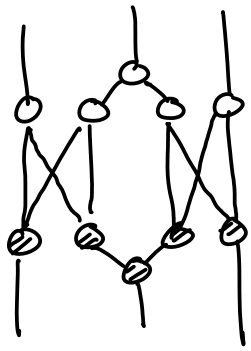
...,or:



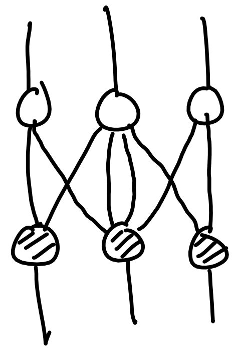
(iii)



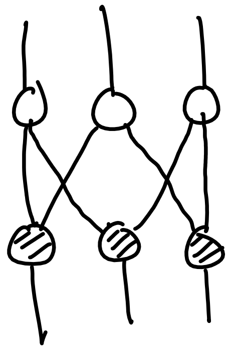
$\overset{SC\times 2}{\approx}$



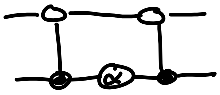
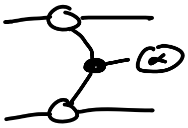
=

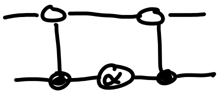
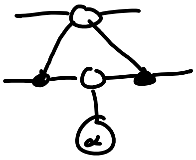
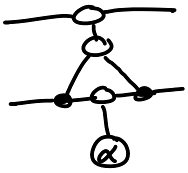


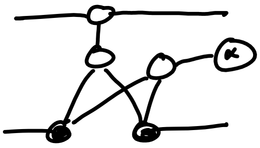
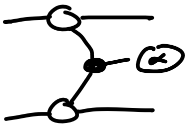
$\overset{C}{\approx}$

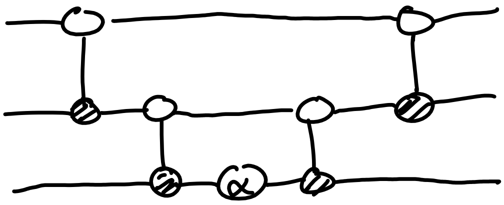
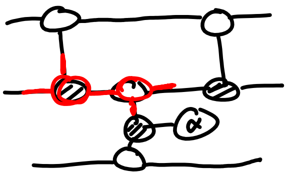


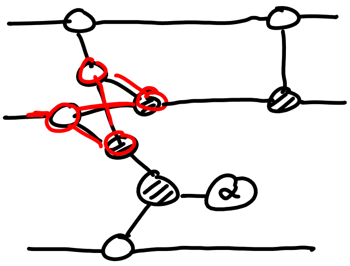
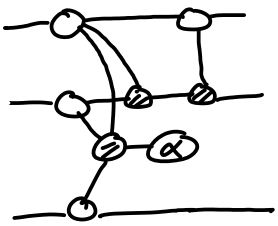
6.3

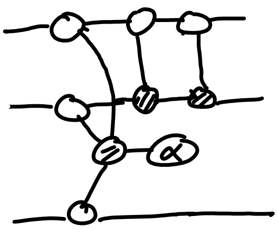
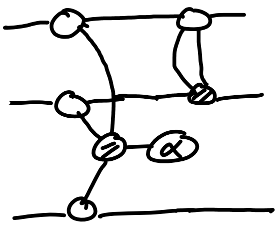
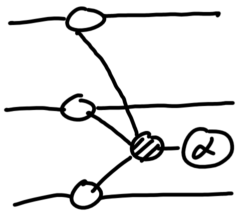
Lem1  $\approx$ 

Proof  $=$  $=$ 

$\overset{\text{OCM}}{=}$  $\overset{\text{SC}}{\approx}$  ∎

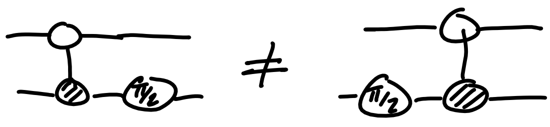
(i)  $\overset{\text{Lem1}}{\approx}$ 

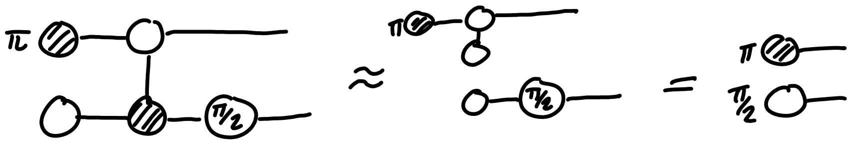
$\overset{\text{SC}}{\approx}$  $=$ 

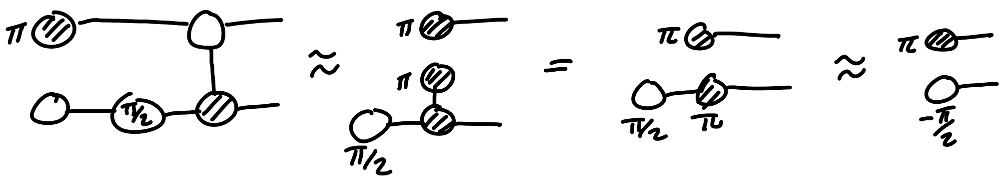
$=$  $=$  $\approx$ 

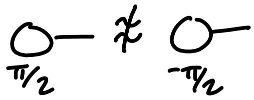
6.3 (cont'd)

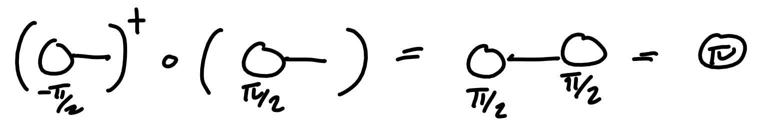
(ii) Need to show:



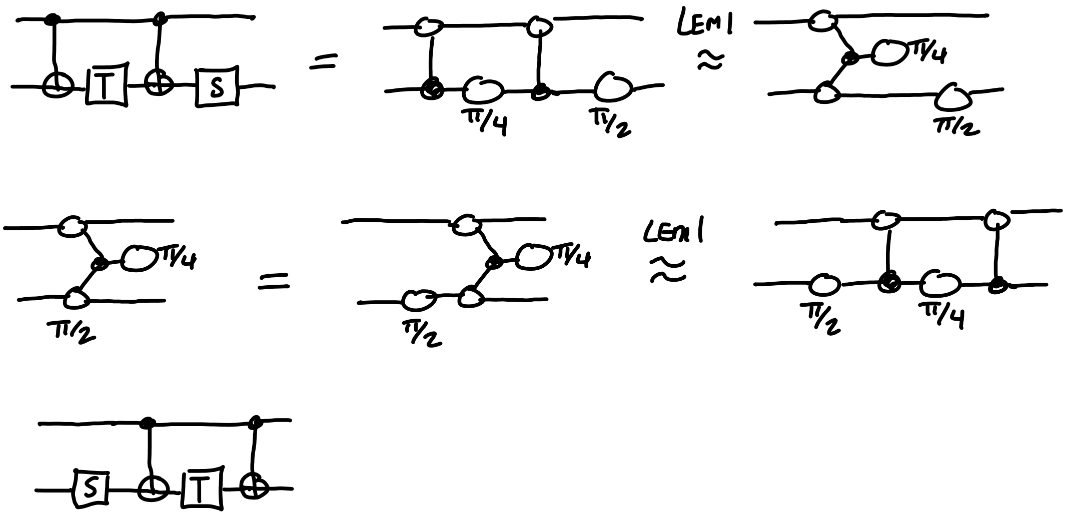




But:  . In fact, they are orthogonal:

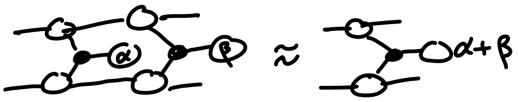
 $= 1 + e^{i\pi} = 0$.

(iii)

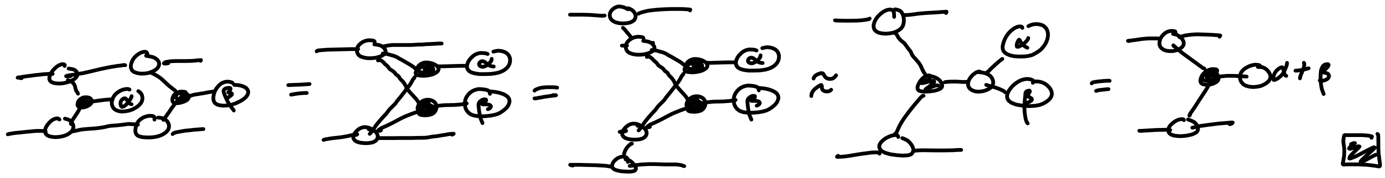


6.4

Lem 2 ("gadget" fusion)

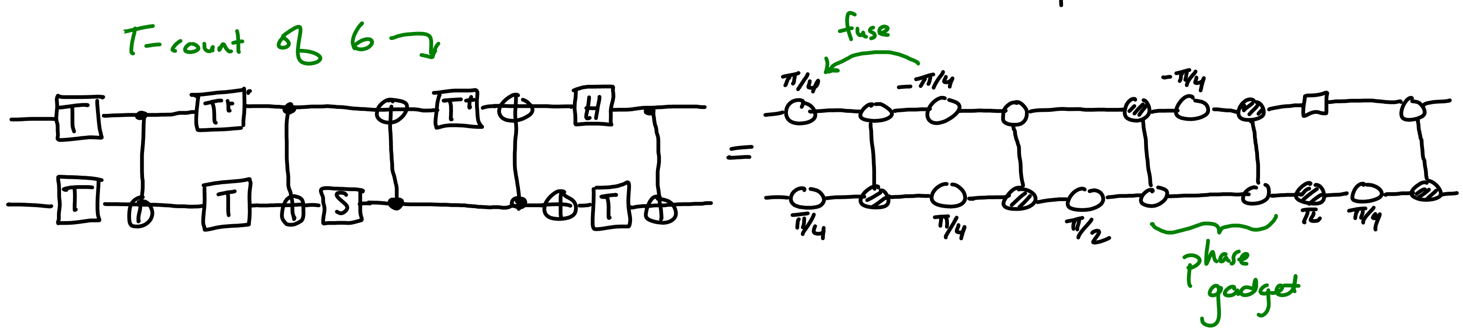


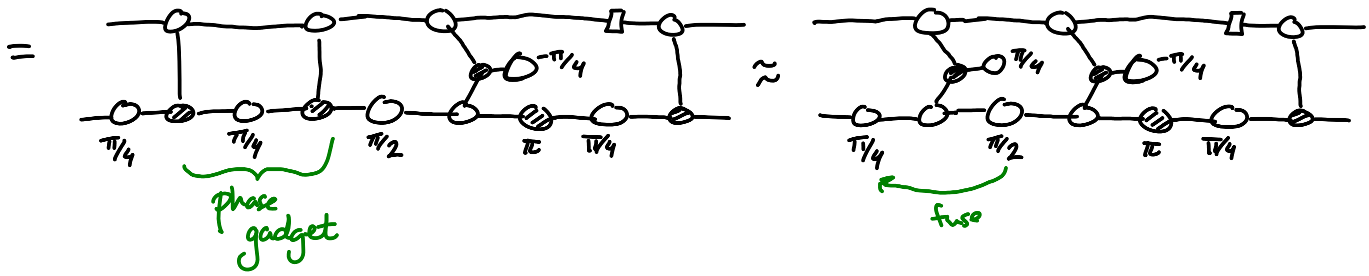
Pf

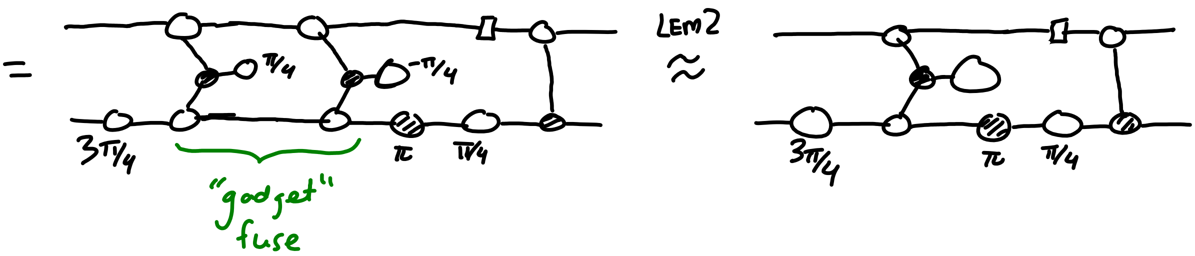


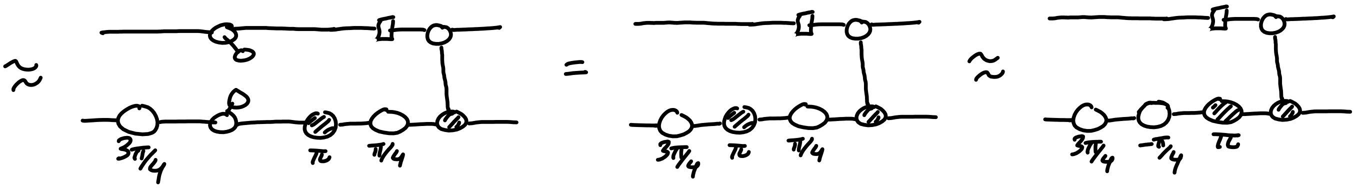
There is not a unique solution. Here's a possibility:

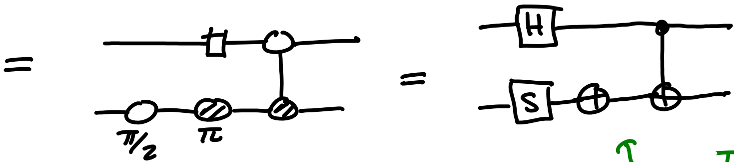
T-count of 6











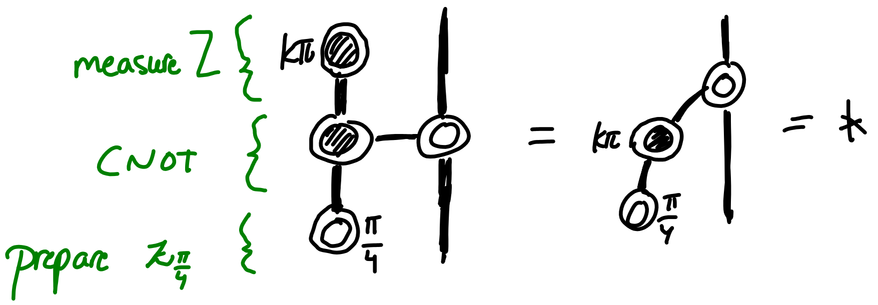
T-count of 0!

6.5

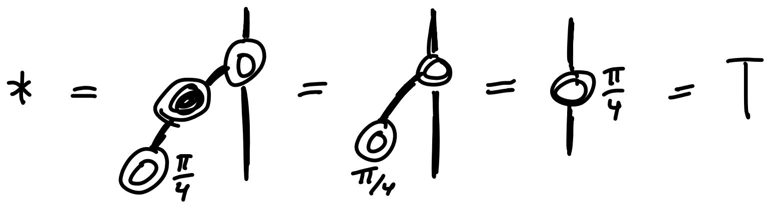
Note: in this question, we write non-deterministic processes using indices, rather than cq-maps / classical wires.

E.g. a Z ONB measurement  is written as the non-deterministic process $\Big($  $\Big)^k \approx \Big($  $\Big)^k$.

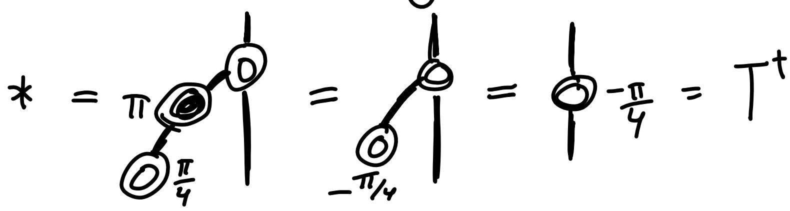
(a)



— If measurement gives outcome $k=0$:

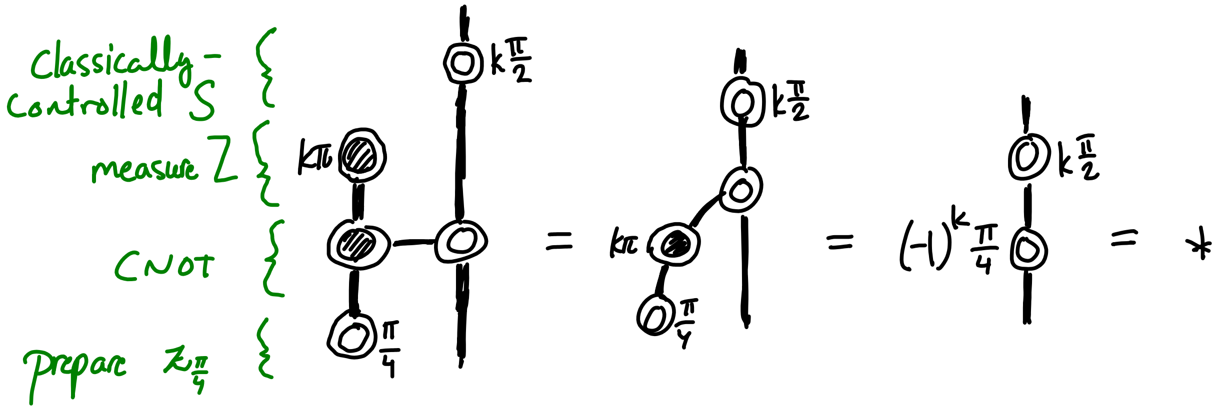


— If measurement gives outcome $k=1$:

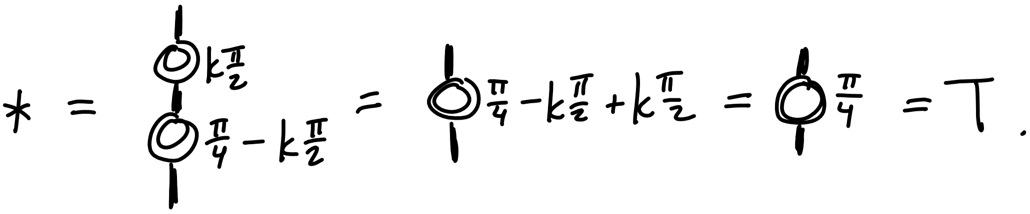


6.5 (cont'd)

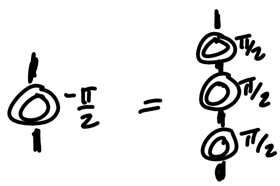
(b)



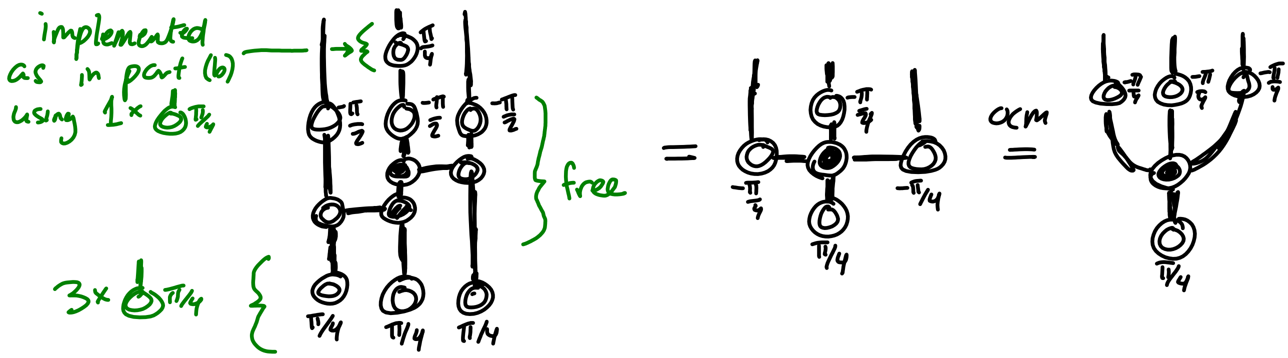
n.b. $(-1)^k\frac{\pi}{4} = \frac{\pi}{4} - k\frac{\pi}{2}$ for $k \in \{0,1\}$



(c) We already know we can convert 1 $\frac{\pi}{4}$ into 1 $\frac{\pi}{4}$ using free operations. We can also make $S^\dagger$ using free op's:



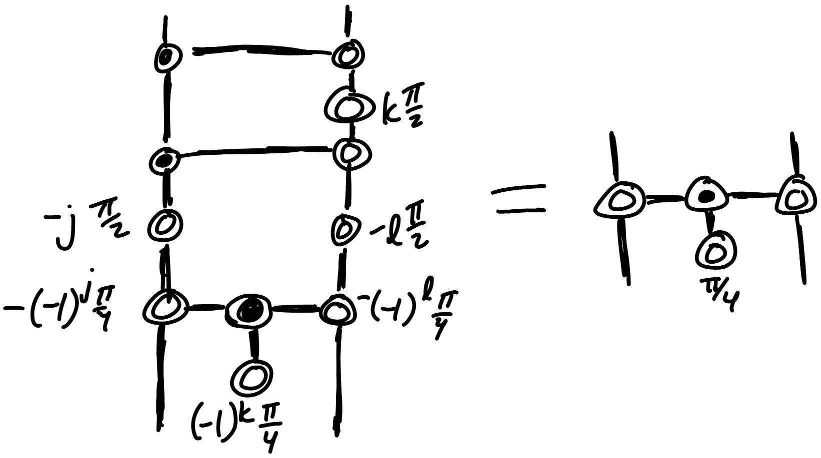
So, with $4\times$ $\pi/4$ and free ops:



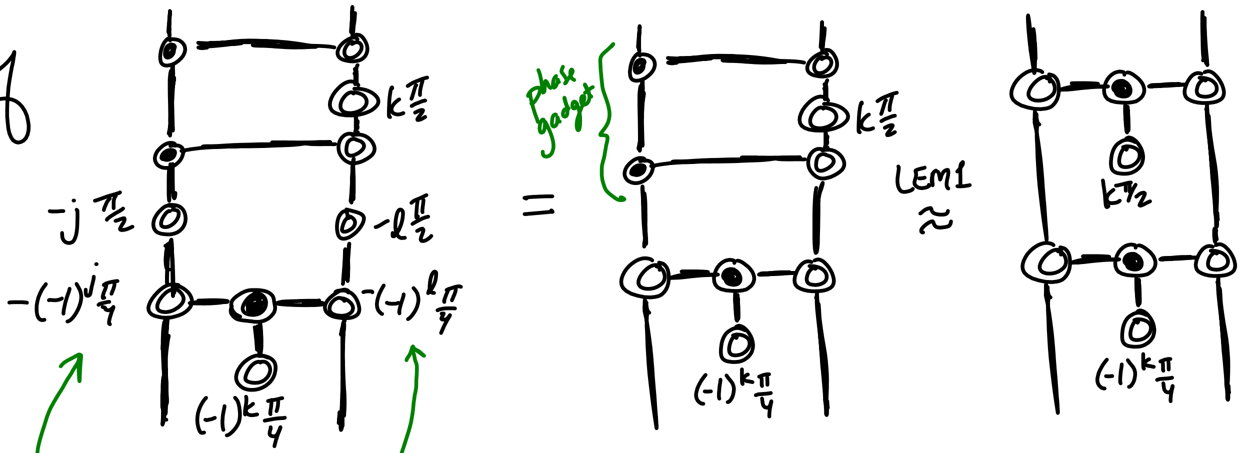
6.5 (cont'd)

(d)

LEM 3

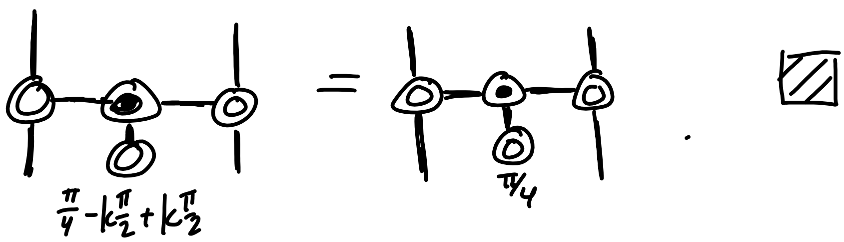


Proof



$$-(-1)^x\frac{\pi}{4} = -\frac{\pi}{4} + x\frac{\pi}{2}$$

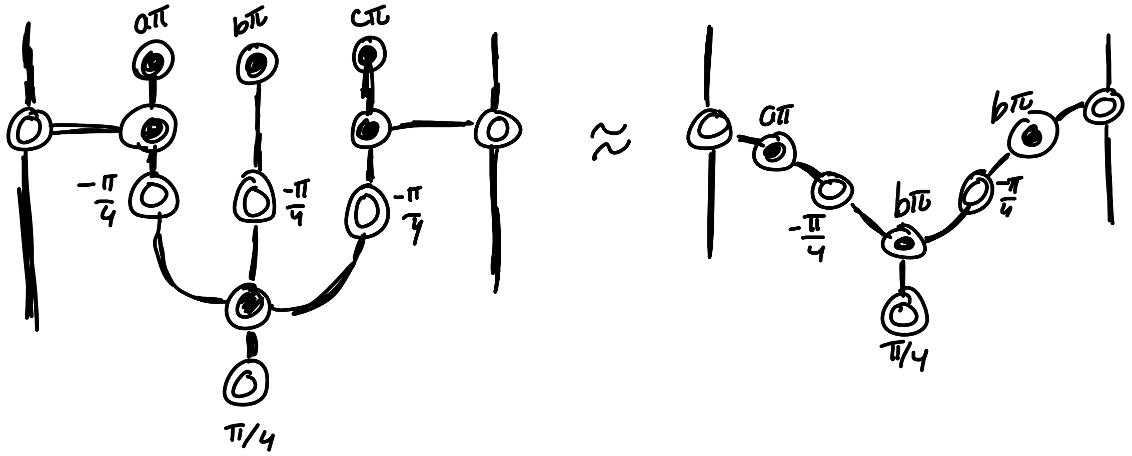
LEM2 $\approx$

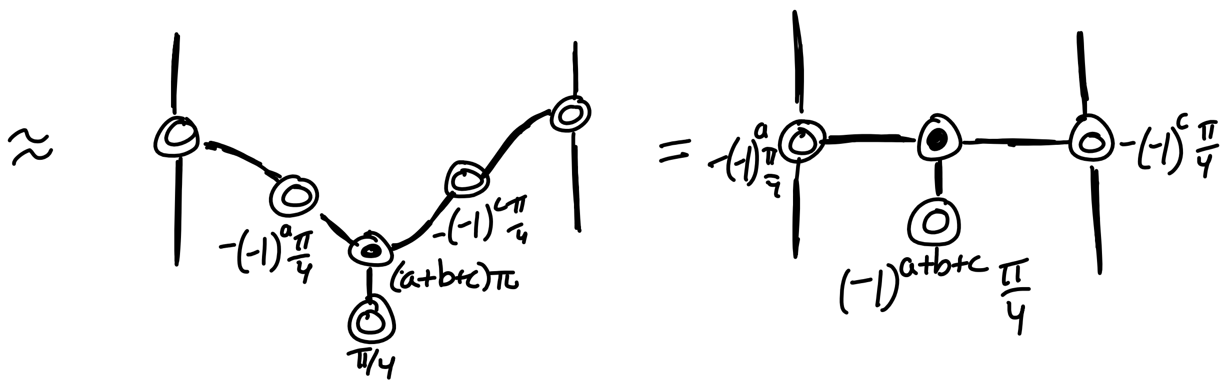


$\square$

6.5 (cont'd)

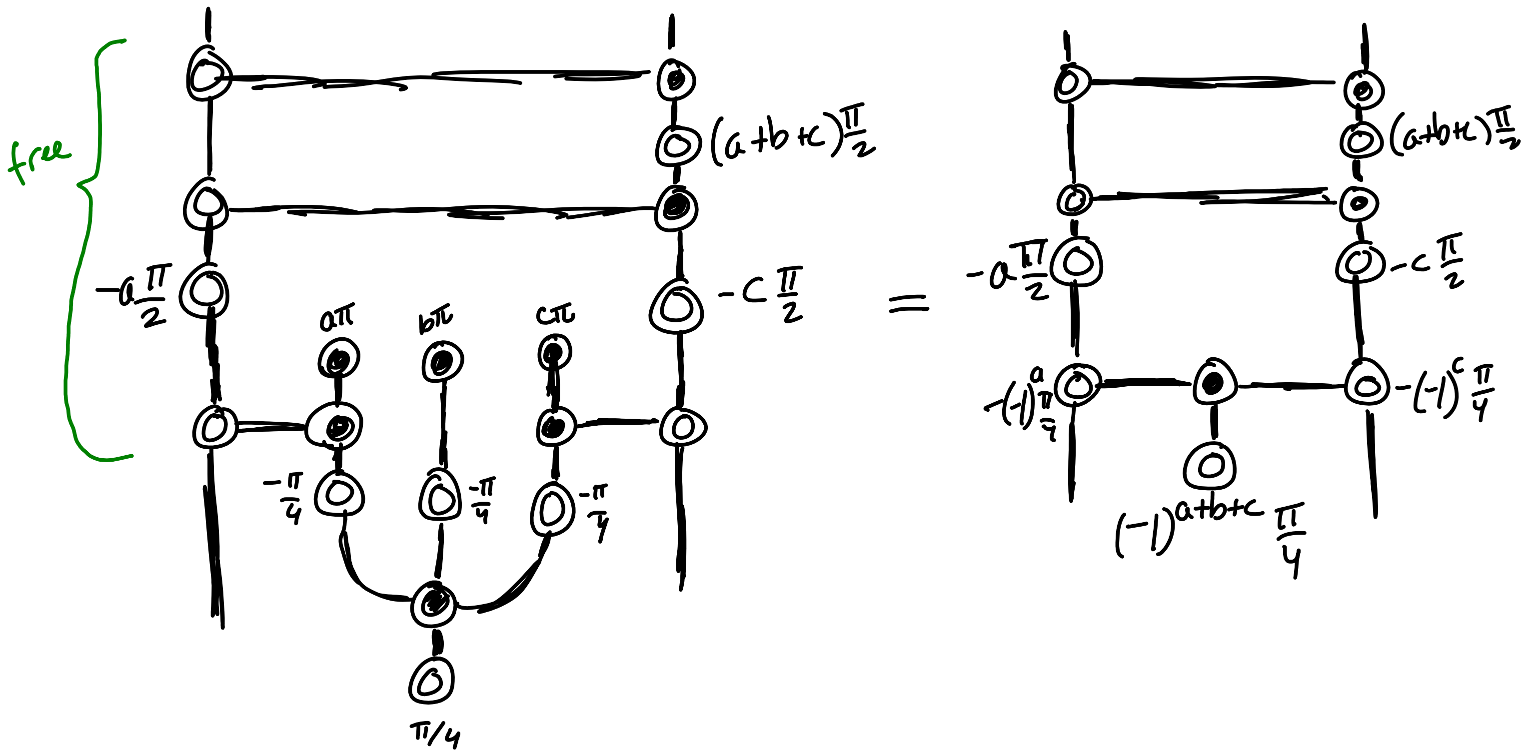
Now, starting with $P_{\frac{\pi}{4}}$, we can get:



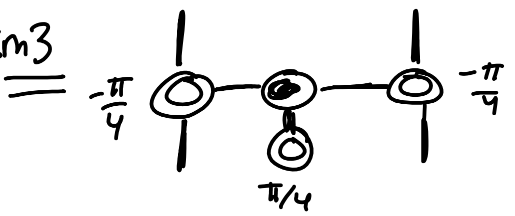


6.5 (cont'd)

Hence, we can apply LEM 3, for $j=a$, $k=a+b+c$, $l=c$:



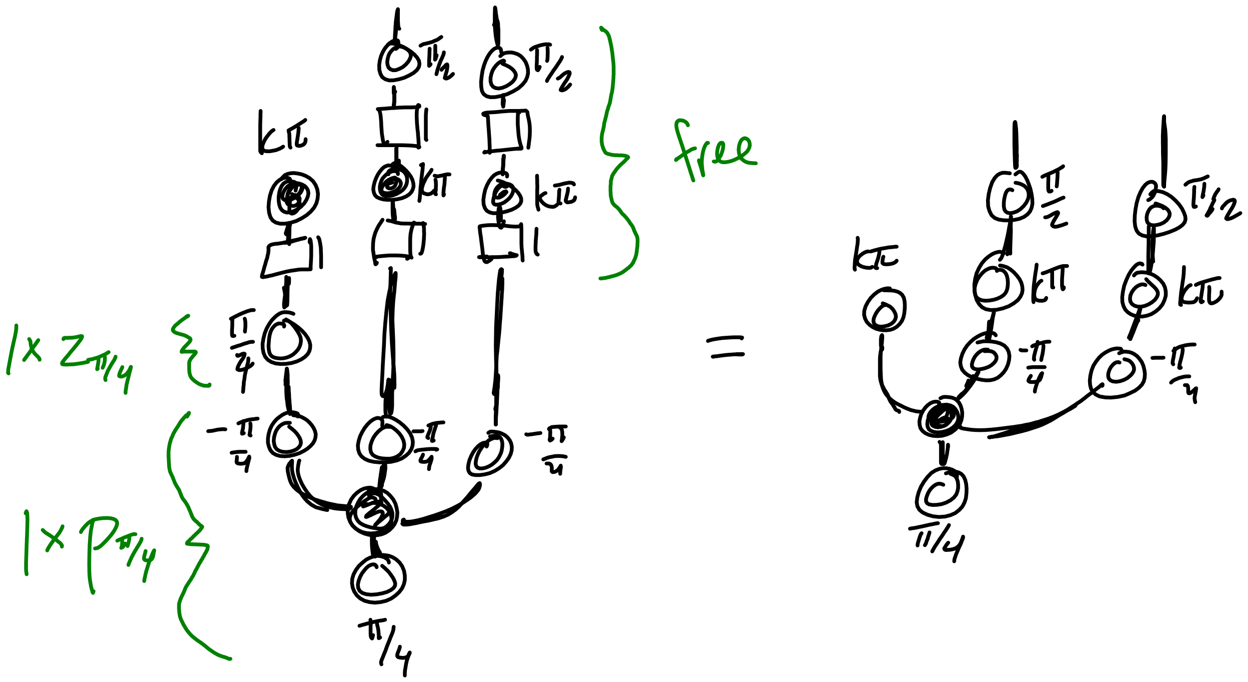
LEM3



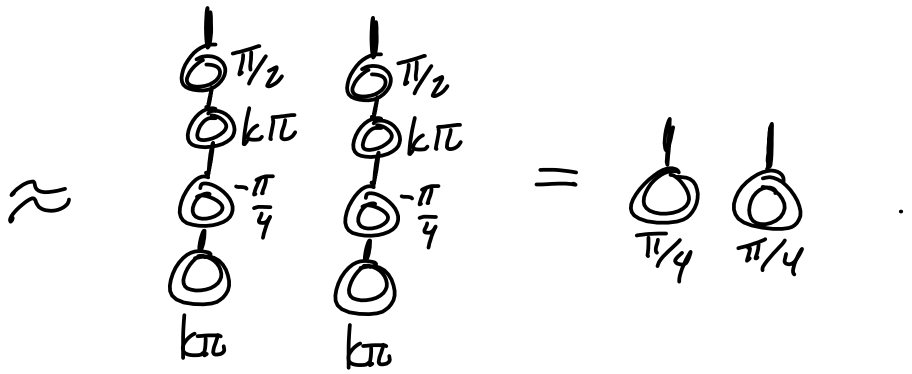
$= CS$.

6.5 (cont'd)

(e)



kπ
π/2
π/2
kπ
kπ
free
1× $Z_{\pi/4}$
π/4
−π/4
−π/4
−π/4
1× $P_{\pi/4}$
π/4
=
kπ
π/2
π/2
kπ
kπ
−π/4
−π/4
π/4



≈
π/2
π/2
kπ
kπ
−π/4
−π/4
kπ
kπ
=
π/4
π/4