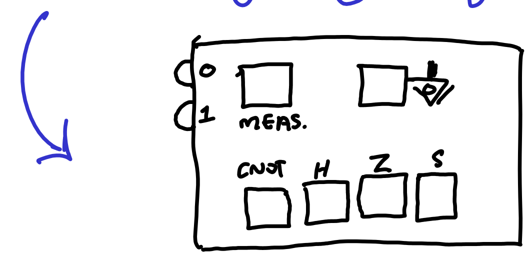
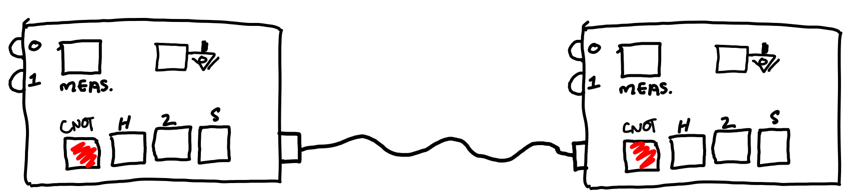
Chapter 11: Quantum Foundations

* Suppose I have a box that looks like this, which I bought for $10,000:

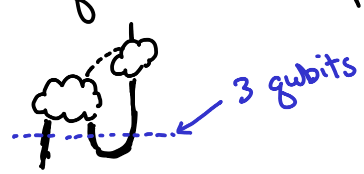
"Totally legit qubit™!!"



* .... actually I bought 3 of them (for $30,000!)

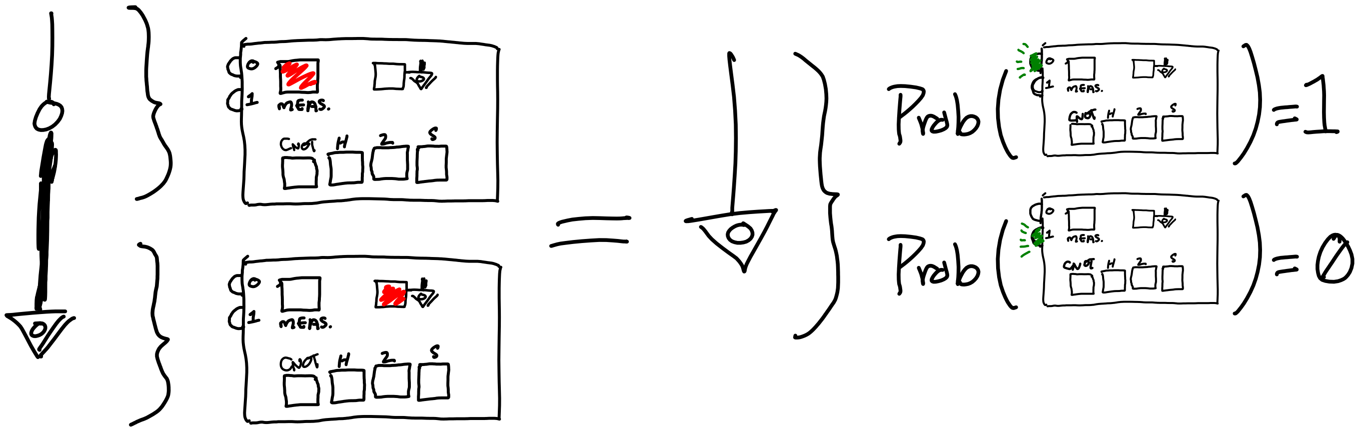


so I can do quantum teleportation:

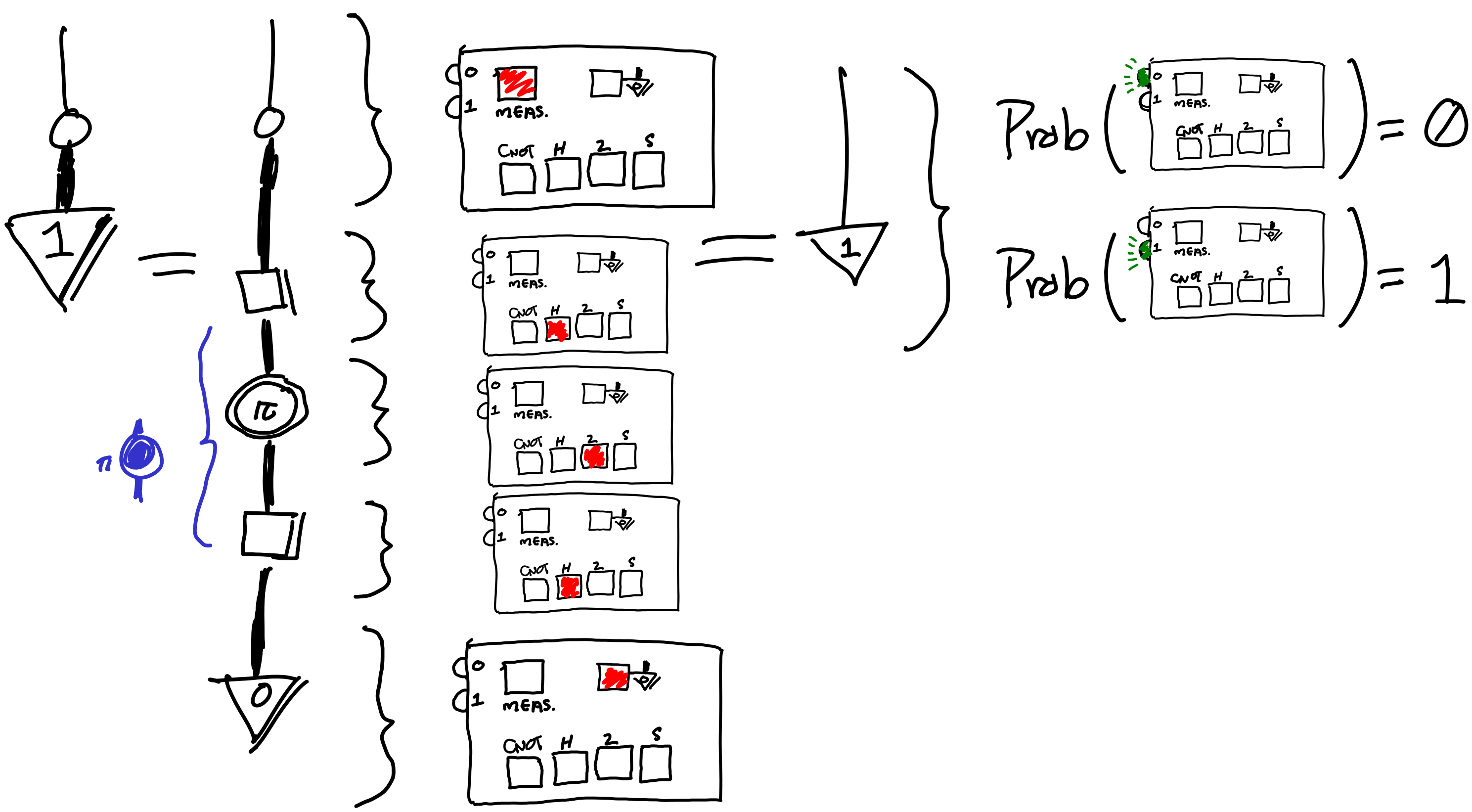


Q: Did I get ripped off?

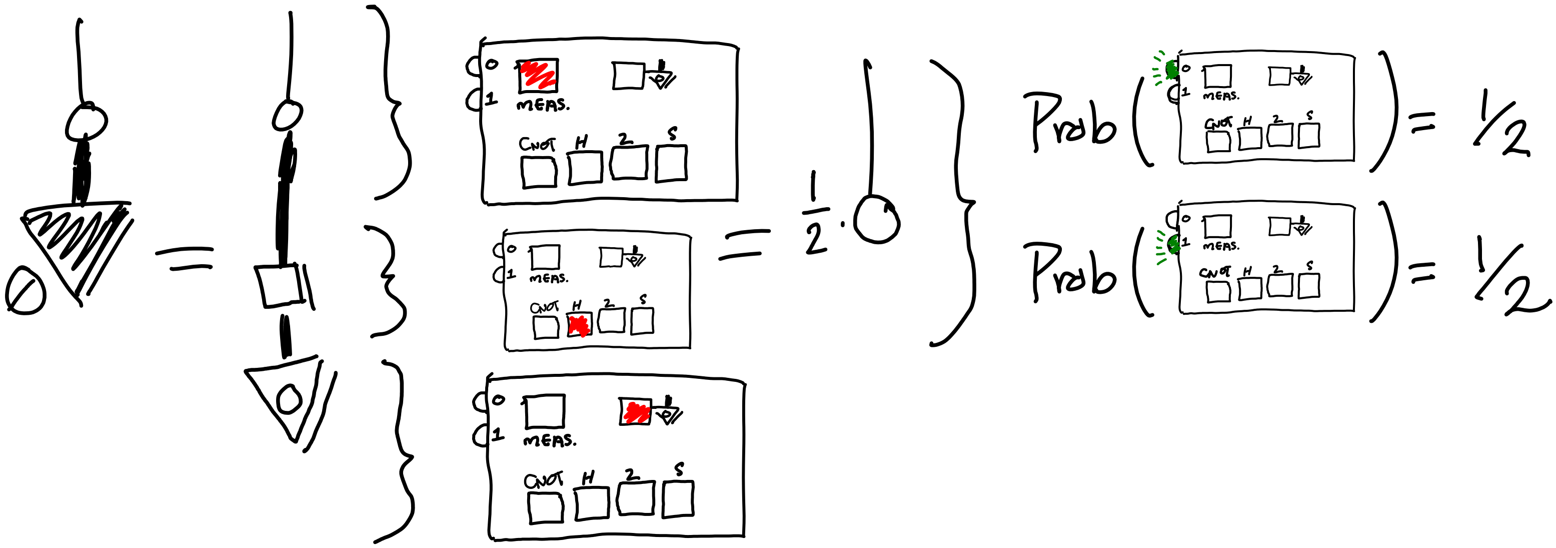
Test #1 ✓



Test #2 ✓

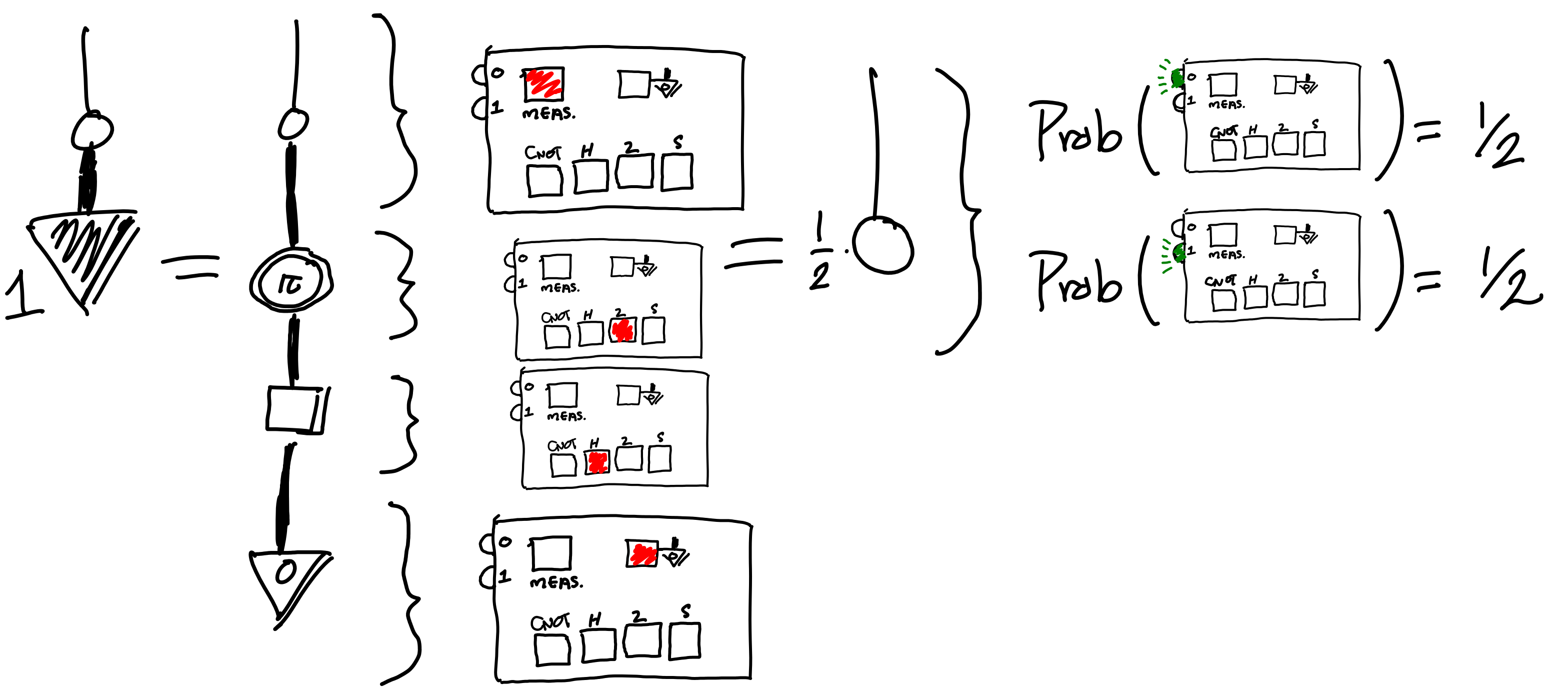


Test #3 ✓



0
=
=
1/2 ·
MEAS.
CNOT H Z S
Prob( ) = 1/2
Prob( ) = 1/2

Test #4 ✓

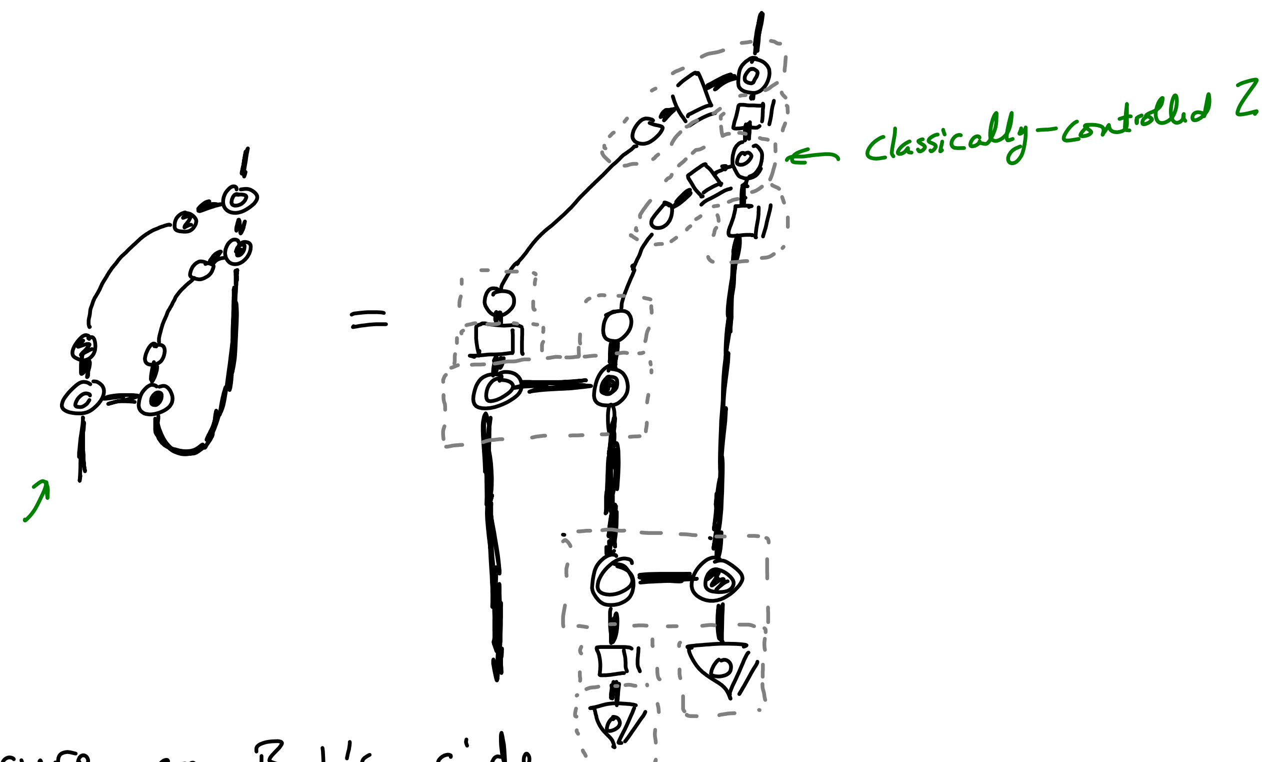


1
=
π
=
1/2 ·
MEAS.
CNOT H Z S
Prob( ) = 1/2
Prob( ) = 1/2

Test #5

1. Prepare {  }

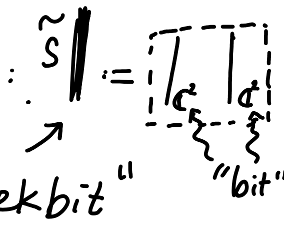
2. Teleport to Bob:



3. Measure on Bob's side.

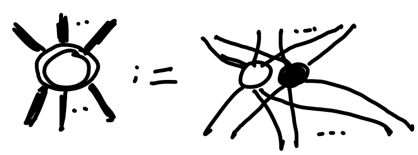
⟹ still looks quantum!

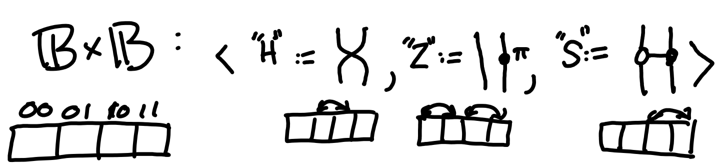
Q: Did I get ripped off?

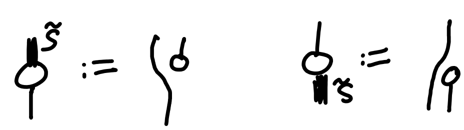
The trick: use 2 classical bits: 

— R. Spekkens. "In defense of an epistemic view on quantum states: a toy theory."

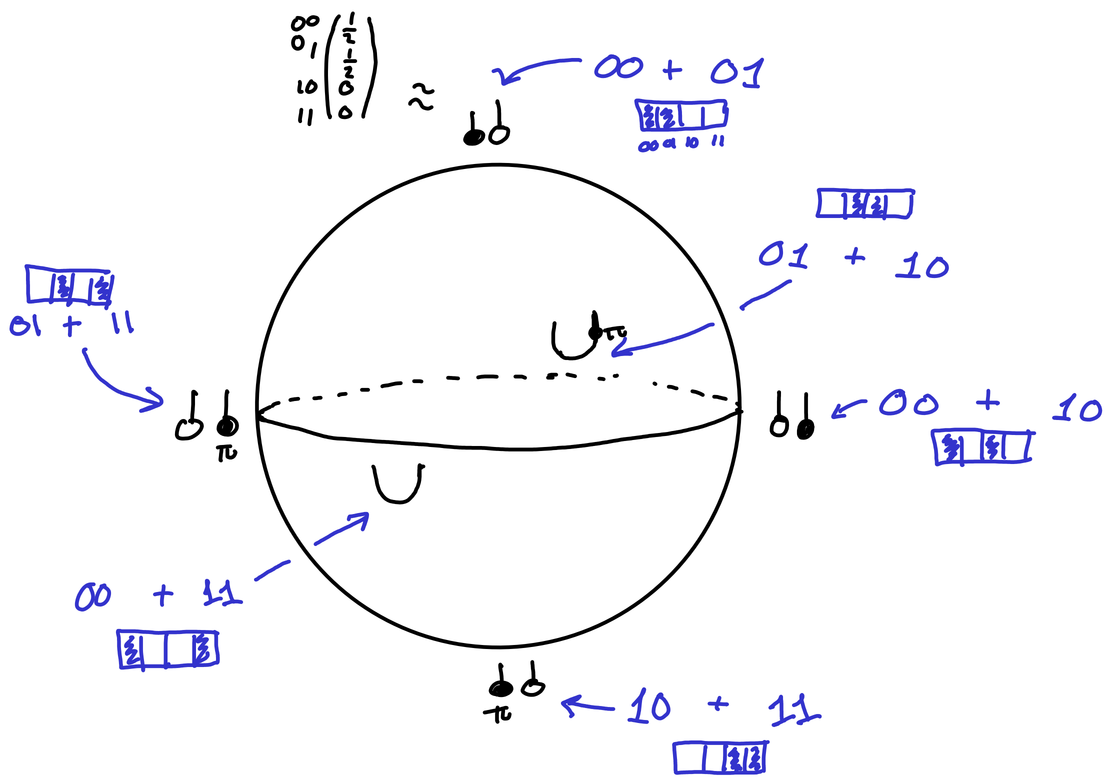
<u>DEF</u> The process theory <u>spek</u> $\subseteq$ <u>classical maps</u> is generated by:

* "spek spiders"  (← n.b. this is <u>not</u> a quantum map)

* all permutations on $\mathbb{B}\times\mathbb{B}$: 

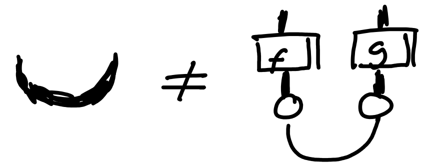
* "encoding" & "measurement" 

States for a single system ⇝ The Speksphere:

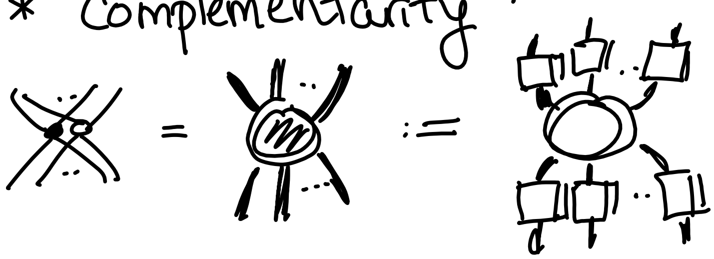


Spek is classical by design, but it has:

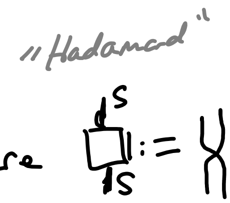
* entanglement:



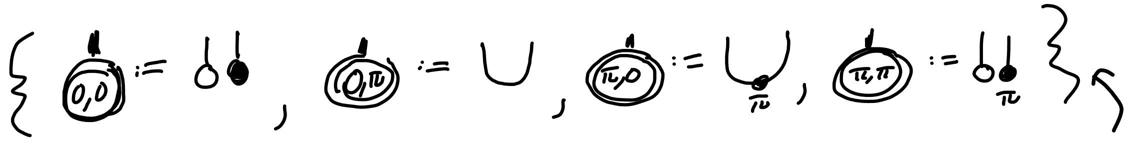
* (strong) complementarity:



where



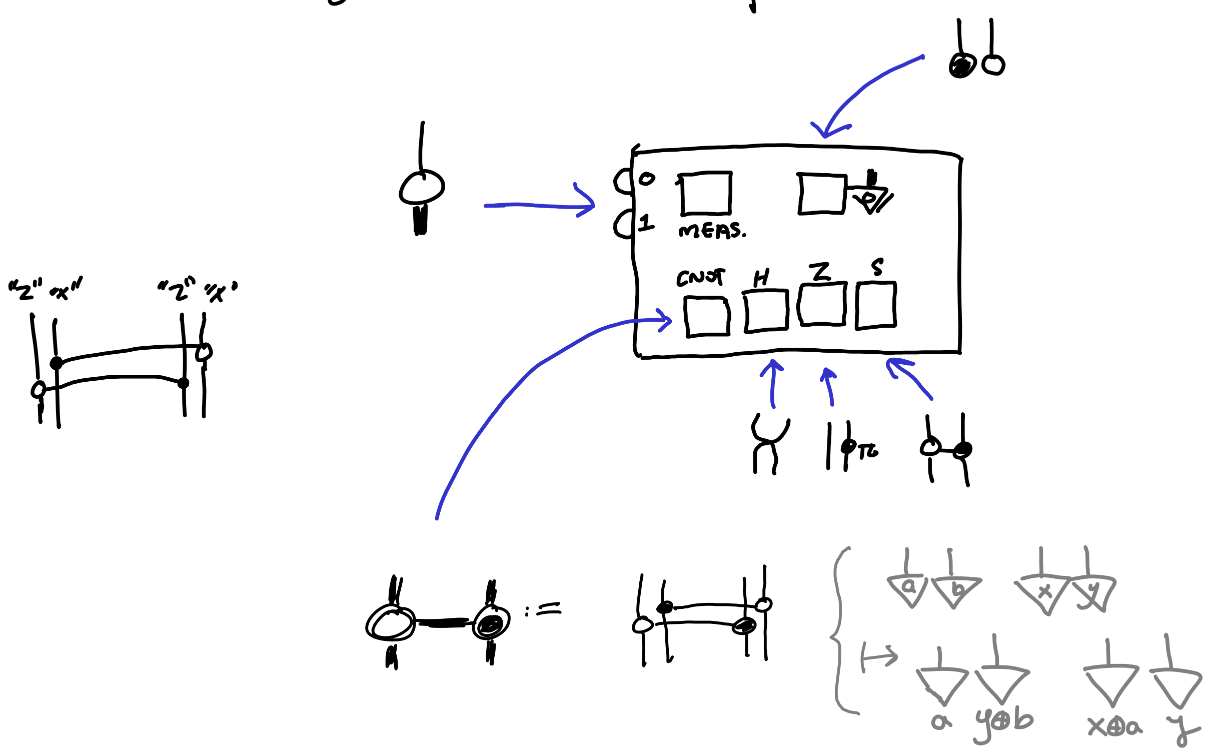
* phase groups



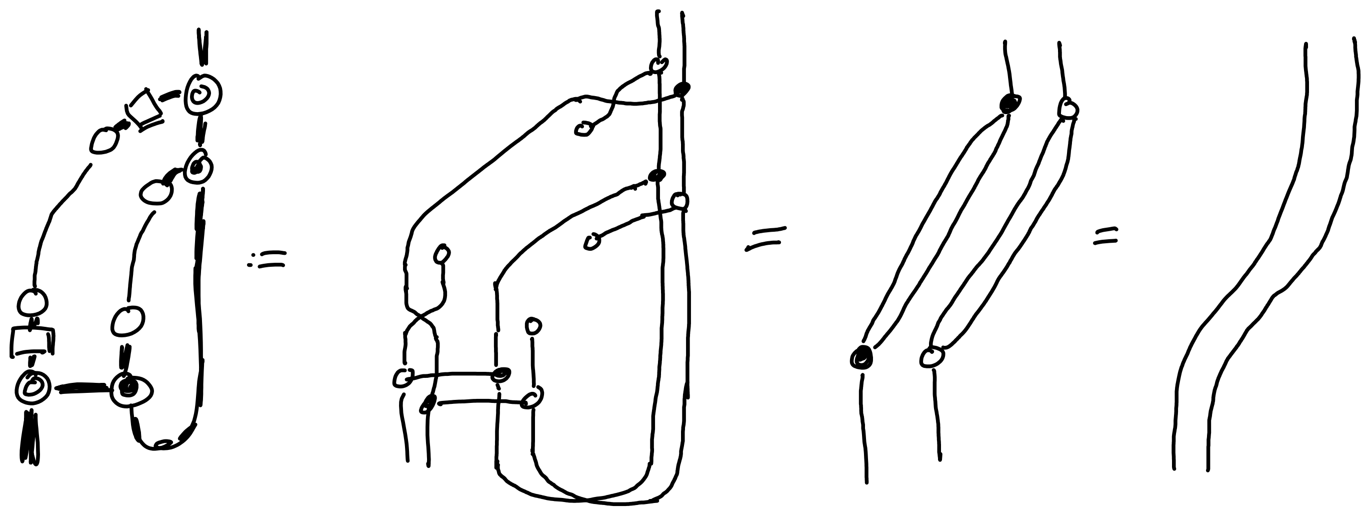
$\cong \mathbb{Z}_2 \times \mathbb{Z}_2$

not $\mathbb{Z}_4$

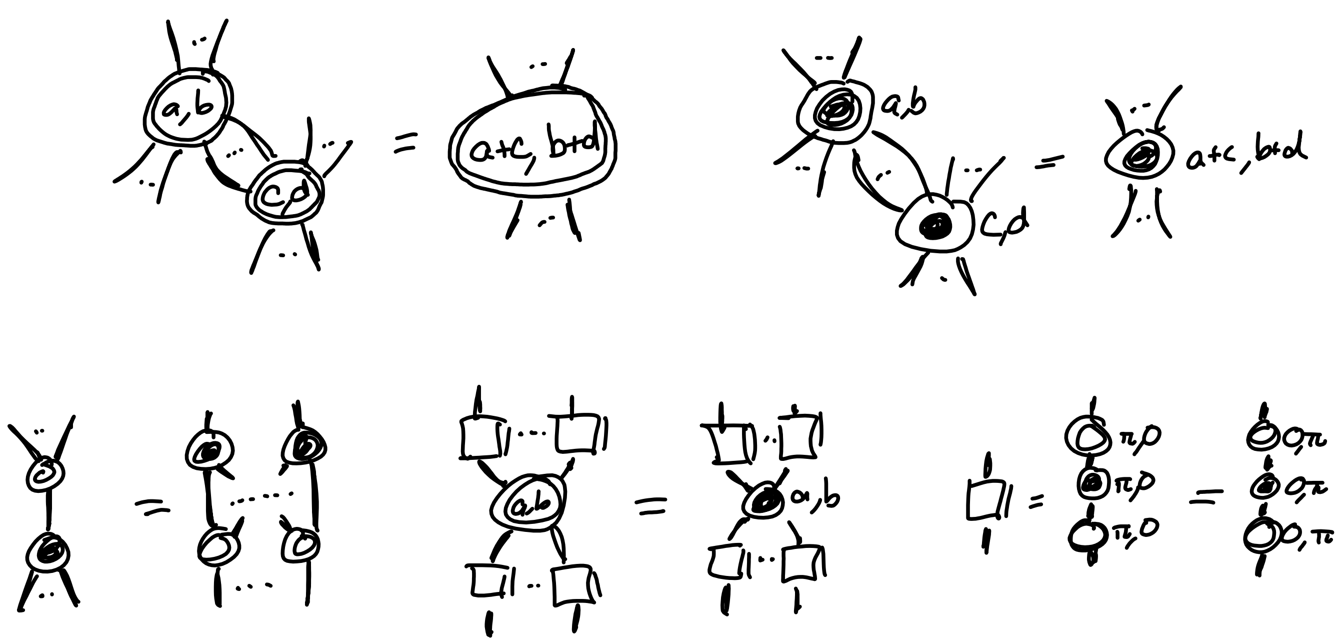
* "quantum-like" processes



* teleportation:



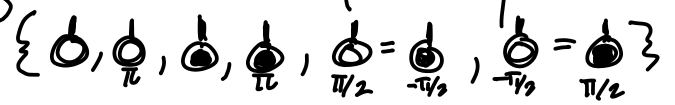
* ... and a complete "spek-ZX" calculus:



clifford maps ⊆ quantum maps

generated by:

* a family of spiders  $\hat{\mathbb{C}}^2$

* all rotations of the Bloch sphere preserving the 6 states 

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spek ⊆ classical maps (probabilistic version)

OR

spek′ ⊆ relations (possibilistic version, in PQP)

generated by:

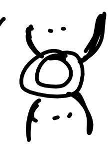
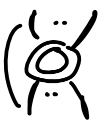
* a family of spiders  $\tilde{S}$

* all transformations of the 6 spek sphere states 

coming from a permutation

Q: What's the difference?

A: The phase group:

Phase group( $\hat{\mathbb{C}}^2$) $\cong \mathbb{Z}_4$ Phase gp. ( $\tilde{S}$) $\cong \mathbb{Z}_2 \times \mathbb{Z}_2$

<u>Clifford maps</u>    $\pi/2$ <u>spek</u>    $0,\pi$ / $\pi,0$

Q: Is it a big deal?

A: Yes! $\mathbb{Z}_4$ can be used to prove quantum nonlocality!

The GHZ/Mermin Argument 11.1.2