Probabilistic Model Checking and Strategy Synthesis

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Overview

- Probabilistic model checking
  - verification vs. strategy synthesis
  - Markov decision processes (MDPs)
  - example: robot navigation
- Multi-objective probabilistic model checking
  - examples: power management/team-formation
- Stochastic (multi-player) games
  - example: energy management

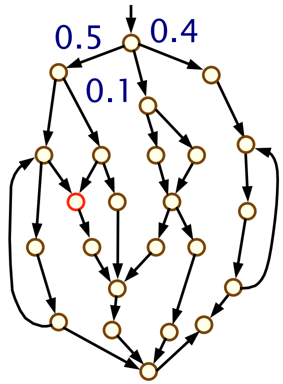
Motivation

- Verifying probabilistic systems…
  - unreliable or unpredictable behaviour
    - failures of physical components
    - message loss in wireless communication
    - unreliable sensors/actuators
  - randomisation in algorithms/protocols
    - random back-off in communication protocols
    - random routing to reduce flooding or provide anonymity

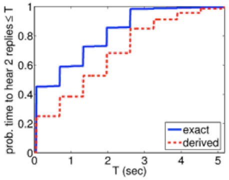
- We need to verify quantitative system properties
  - “the probability of the airbag failing to deploy within 0.02 seconds of being triggered is at most 0.001”
  - not just correctness: reliability, timeliness, performance, …
  - not just verification: correctness by construction

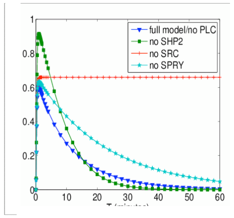
Probabilistic model checking

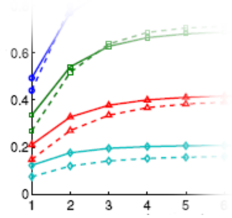
- Construction and analysis of probabilistic models
  - state-transition systems labelled with probabilities (e.g. Markov chains, Markov decision processes)
  - from a description in a high-level modelling language

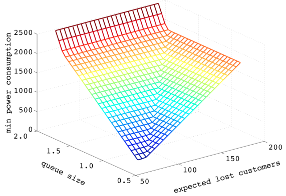


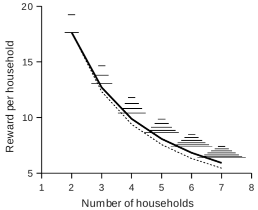
- Properties expressed in temporal logic, e.g. PCTL:
  - trigger → $P_{\geq 0.999}$ [$F^{\leq 20}$ deploy]
  - "the probability of the airbag deploying within 20ms of being triggered is at at least 0.999"
  - properties checked against models using exhaustive search and numerical computation











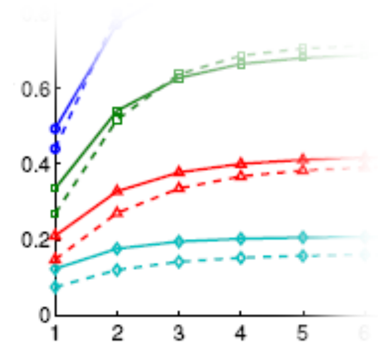
Probabilistic model checking

- Many types of probabilistic models supported
- Wide range of quantitative properties, expressible in temporal logic (probabilities, timing, costs, rewards, …)
- Often focus on numerical results (probabilities etc.)
  - analyse trends, look for system flaws, anomalies

- $P_{\leq 0.1}$ [F *fail*] – "the probability of a failure occurring is at most 0.1"

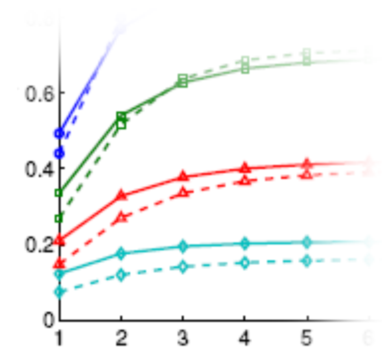
↓

- $P_{=?}$ [F *fail*] – "what is the probability of a failure occurring?"



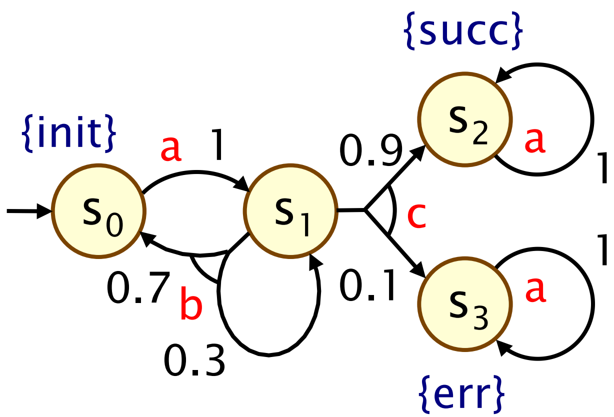
Probabilistic model checking

- Many types of probabilistic models supported
- Wide range of quantitative properties, expressible in temporal logic (probabilities, timing, costs, rewards, …)
- Often focus on numerical results (probabilities etc.)
  - analyse trends, look for system flaws, anomalies
- Provides "exact" numerical results/guarantees
  - compared to, for example, simulation
- Combines numerical & exhaustive analysis
  - especially useful for nondeterministic models
- Fully automated, tools available, widely applicable
  - network/communication protocols, security, biology, robotics & planning, power management, …



Markov decision processes (MDPs)

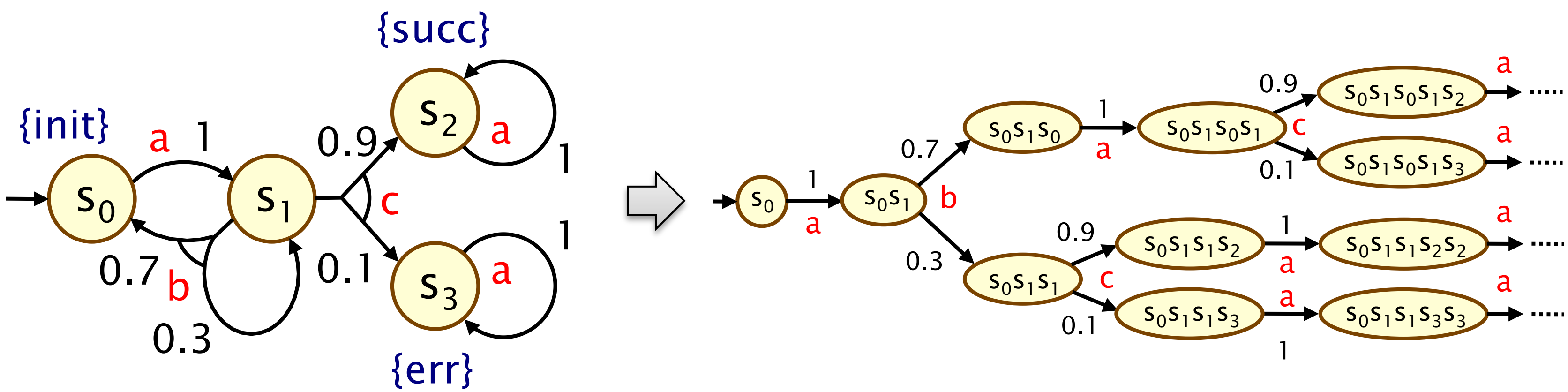
- Markov decision processes (MDPs)
  - widely used also in: AI, planning, optimal control, …
  - model nondeterministic as well as probabilistic behaviour



- Nondeterminism for:
  - control: decisions made by a controller or scheduler
  - adversarial behaviour of the environment
  - concurrency/scheduling: interleavings of parallel components
  - abstraction, or under-specification, of unknown behaviour

Strategies

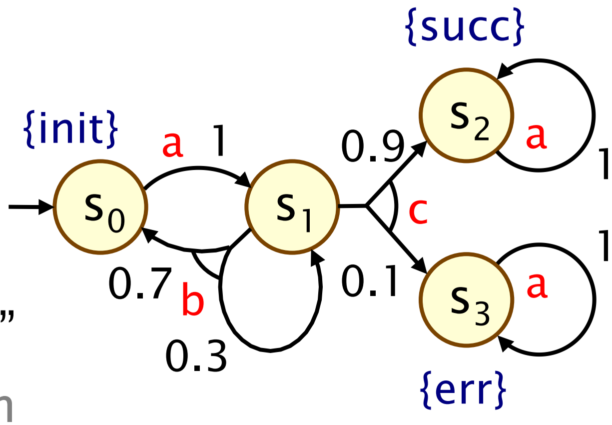
- A strategy (or "policy" or "adversary")
  - is a resolution of nondeterminism, based on history
  - i.e. a mapping from finite paths to (distributions over) actions
  - induces (infinite-state) Markov chain (and probability space)



- Classes of strategies:
  - memory: memoryless, finite-memory, or infinite-memory
  - randomisation: deterministic or randomised

Verification vs. Strategy synthesis

- 1. Verification
  - quantify over all possible strategies (i.e. best/worst-case)
  - $P_{\leq 0.1}$ [F *err*] : “the probability of an error occurring is ≤ 0.1 for all strategies”
  - applications: randomised communication protocols, randomised distributed algorithms, security, …

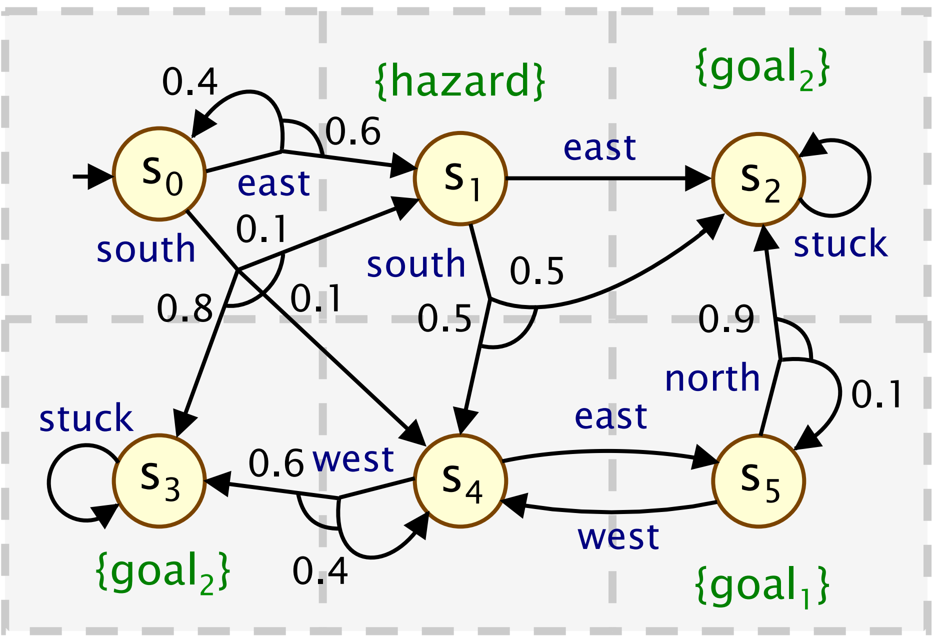


- 2. Strategy synthesis
  - generation of "correct-by-construction" controllers
  - $P_{\leq 0.1}$ [F *err*] : "does there exist a strategy for which the probability of an error occurring is ≤ 0.1?”
  - applications: robotics, power management, security, …

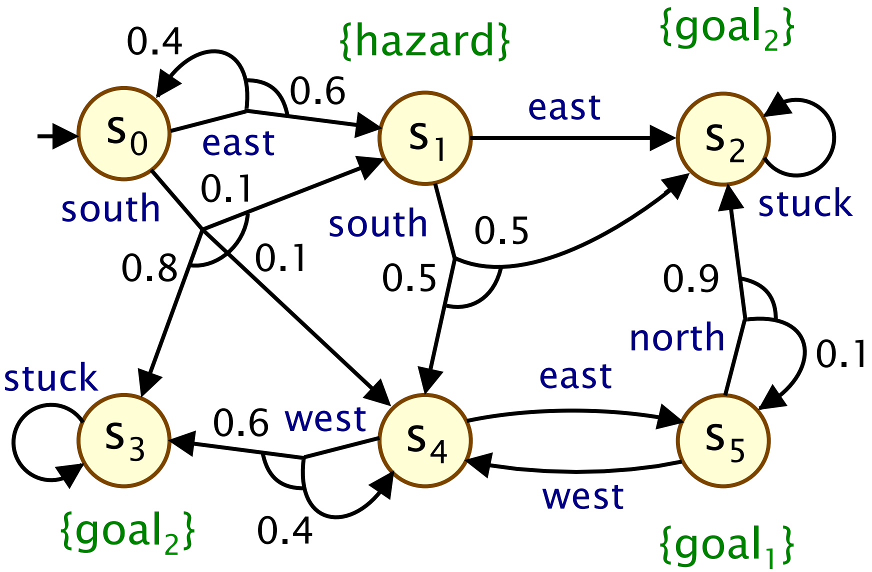
- Two dual problems; same underlying computation:
  - compute optimal (minimum or maximum) values

Running example

- Example MDP
  - robot moving through terrain divided in to 3 x 2 grid



Example – Reachability



Verify: $P_{\leq 0.6}$ [F $goal_1$]

or

Synthesise for: $P_{\geq 0.4}$ [F $goal_1$]

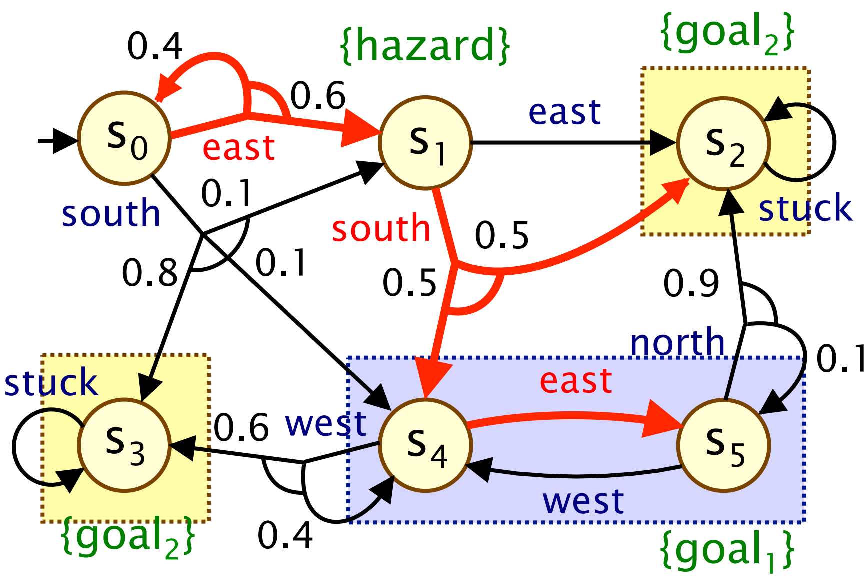
⇓

Compute: $P_{max=?}$ [F $goal_1$]

Optimal strategies:
memoryless and deterministic

Computation:
graph analysis + numerical soln.
(linear programming, value iteration, policy iteration)

Example – Reachability



Optimal strategy:
s_0 : east
s_1 : south
s_2 : –
s_3 : –
s_4 : east
s_5 : –

Verify: $P_{\leq 0.6}$ [F $goal_1$]

or

Synthesise for: $P_{\geq 0.4}$ [F $goal_1$]

⇓

Compute: $P_{max=?}$ [F $goal_1$] = 0.5

Optimal strategies:
memoryless and deterministic

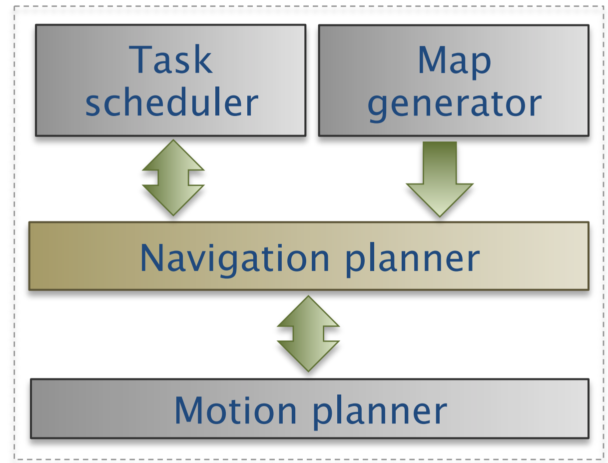
Computation:
graph analysis + numerical soln. (linear programming, value iteration, policy iteration)

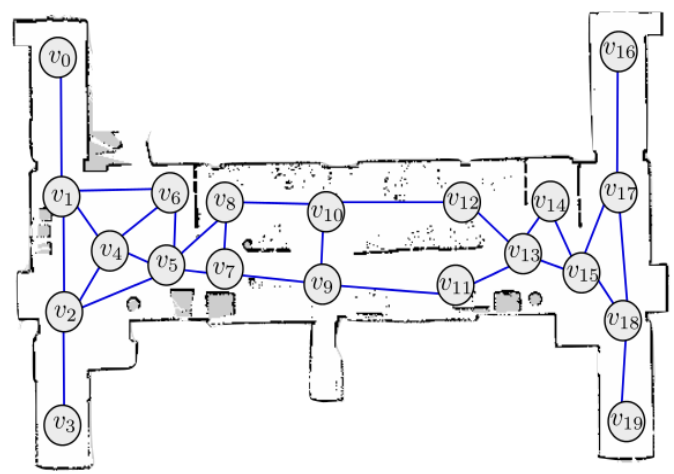
MDPs – Other properties

- Costs and rewards (expected, accumulated values)
  - e.g. $R_{min=?}$ [F $goal_2$] – "what is the minimum expected number of moves needed to reach $goal_2$?"
  - optimal strategies: memoryless and deterministic
  - similar computation to probabilistic reachability
- Probabilistic LTL (multiple temporal operators)
  - e.g. $P_{max=?}$ [(G¬hazard) ∧ (GF $goal_1$)] – "maximum probability of avoiding hazard and visiting $goal_1$ infinitely often?"
  - optimal strategies: finite-memory and deterministic
  - build product MDP, graph analysis, probabilistic reachability
- Expected cost/reward to satisfy (co-safe) LTL formula
  - e.g. $R_{min=?}$ [¬$zone_3$ U ($zone_1$ ∧ (F $zone_4$))] – "minimise exp. time to patrol zones 1 then 4, without passing through 3".

Application: Robot navigation

- Navigation planning: [IROS'14]
  - MDP models navigation through an uncertain environment
  - LTL used to formally specify tasks to be executed
  - synthesise finite-memory strategies to construct plans/controllers
  - links to continuous-space planner

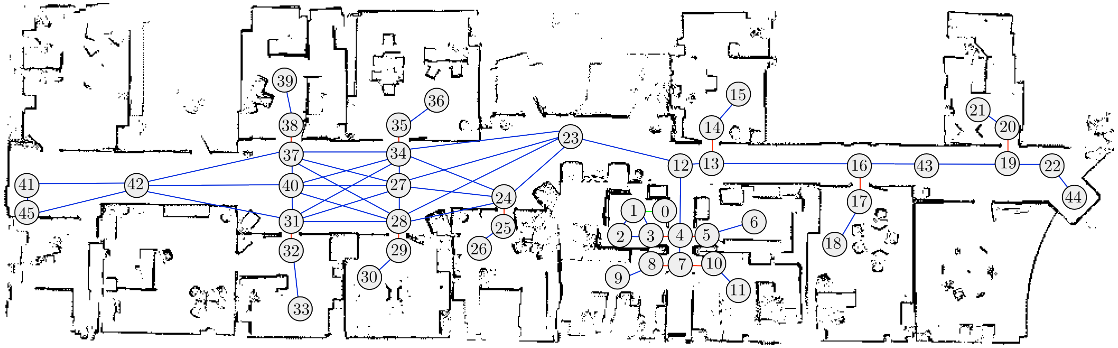




Application: Robot navigation

- Navigation planning MDPs
  - expected timed on edges + probabilities
  - learnt using data from previous explorations

- LTL-based task specification
  - expected time to satisfy (one or more) co-safe LTL formulas
  - c.f. ad-hoc reward structures, e.g. with discounting
  - also: efficient re-planning [IROS'14]; progress metric [IJCAI'15]
- Implementation
  - MetraLabs Scitos A5 robot + ROS module based on PRISM



Overview

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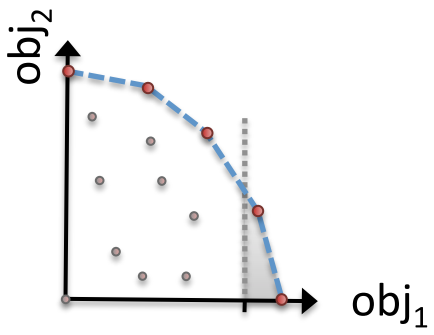
- Multi-objective probabilistic model checking
  - examples: power management/team-formation
- Stochastic (multi-player) games
  - example: energy management

Multi-objective model checking

- Multi-objective probabilistic model checking
  - investigate trade-offs between conflicting objectives
  - in PRISM, objectives are probabilistic LTL or expected rewards
- Achievability queries: multi($P_{>0.95}$ [F *send*], $R^{time}_{>10}$ [C])
  - e.g. "is there a strategy such that the probability of message transmission is > 0.95 and expected battery life > 10 hrs?"
- Numerical queries: multi($P_{max=?}$ [F *send*], $R^{time}_{>10}$ [C])
  - e.g. "maximum probability of message transmission, assuming expected battery life-time is > 10 hrs?"
- Pareto queries:
  - multi($P_{max=?}$ [F *send*], $R^{time}_{max=?}$ [C])
  - e.g. "Pareto curve for maximising probability of transmission and expected battery life-time"



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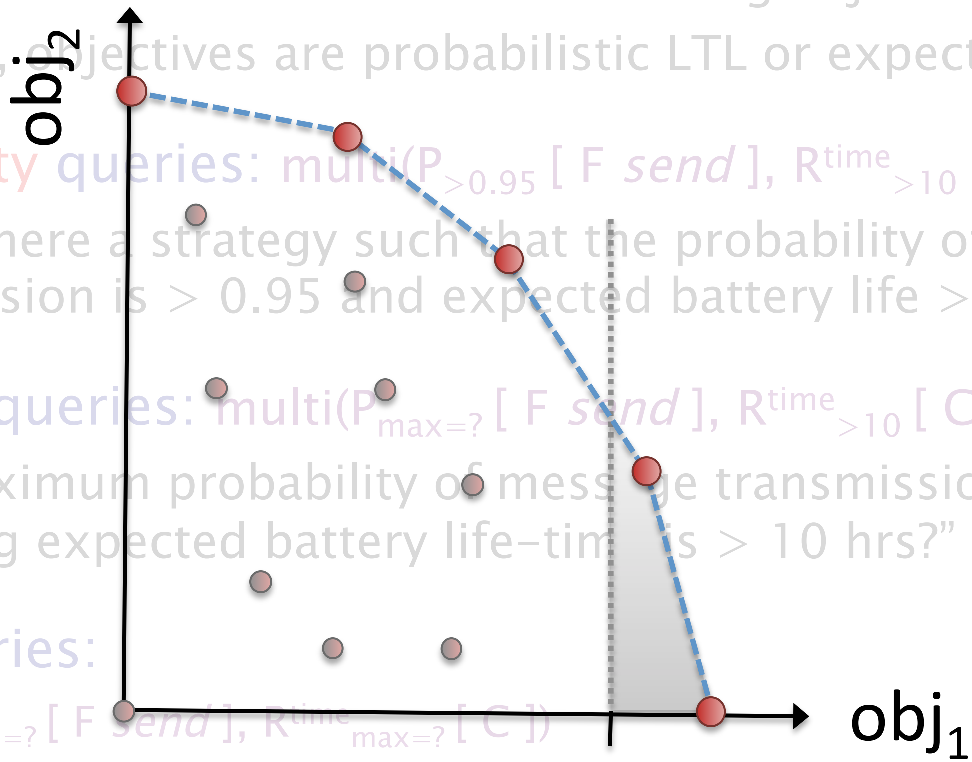

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  - e.g. "Pareto curve for maximising probability of transmission and expected battery life-time”

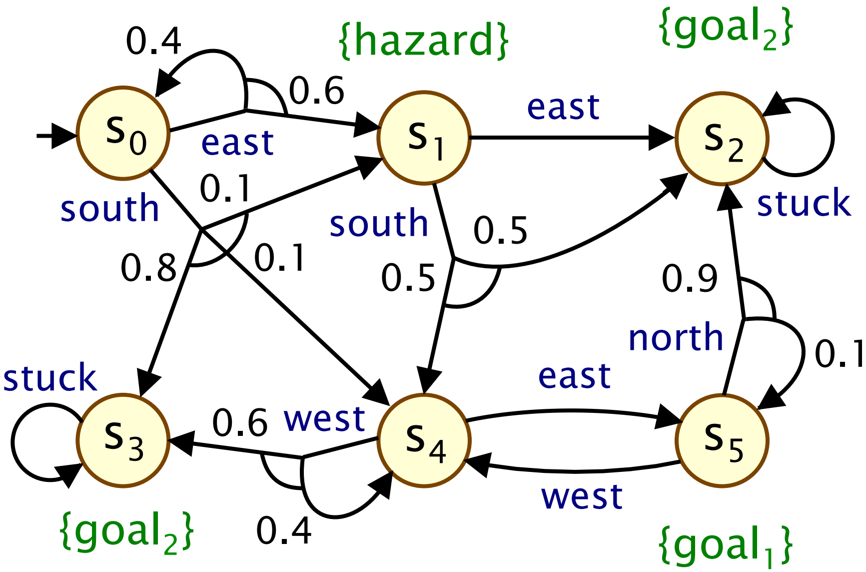
Multi-objective model checking

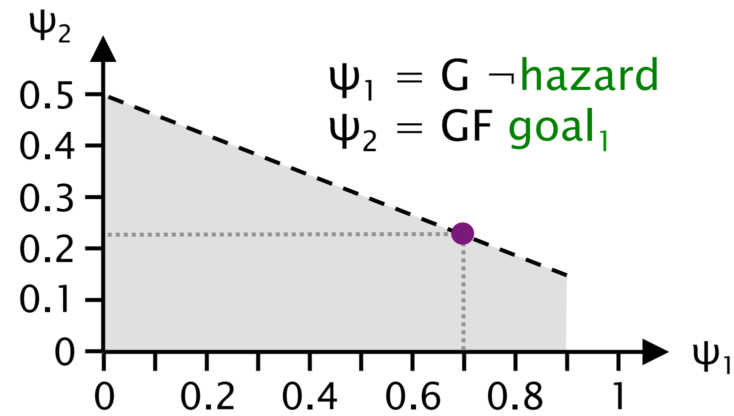
- Optimal strategies:
  - usually finite-memory (e.g. when using LTL formulae)
  - may also need to be randomised

- Computation:
  - construct a product MDP (with several automata), then reduces to linear programming [TACAS'07,TACAS'11]
  - can be approximated using iterative numerical methods, via approximation of the Pareto curve [ATVA'12]

- Extensions [ATVA'12]
  - arbitrary Boolean combinations of objectives
    - e.g. $\psi_1 \Rightarrow \psi_2$ (all strategies satisfying ψ_1 also satisfy ψ_2)
    - (e.g. for assume-guarantee reasoning)
  - time-bounded (finite-horizon) properties

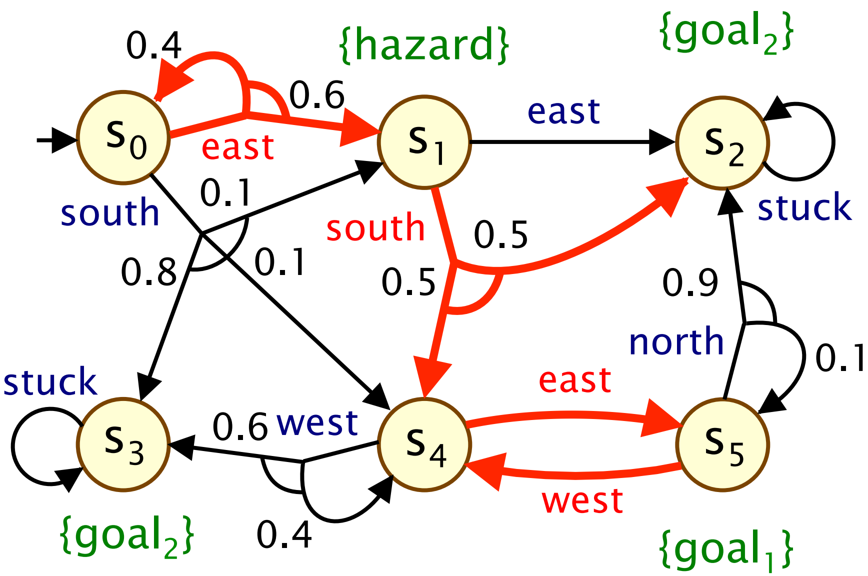
Example – Multi-objective

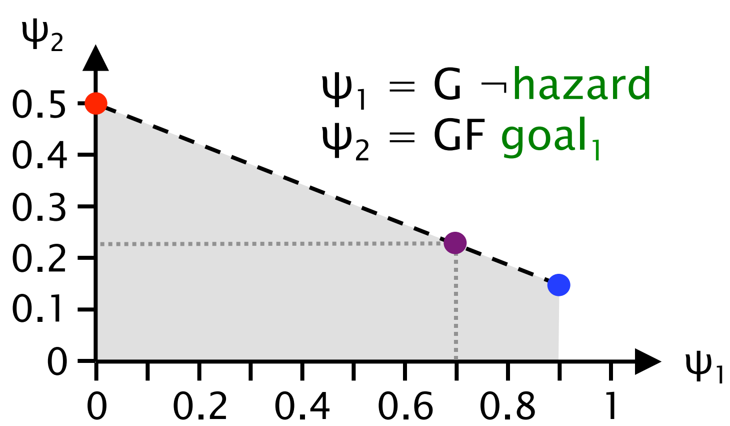




- Achievability query
  - $P_{\geq 0.7}$ [G ¬hazard] ∧ $P_{\geq 0.2}$ [GF $goal_1$] ? True (achievable)
- Numerical query
  - $P_{max=?}$ [GF $goal_1$] such that $P_{\geq 0.7}$ [G ¬hazard] ? ~0.2278
- Pareto query
  - for $P_{max=?}$ [G ¬hazard] ∧ $P_{max=?}$ [GF $goal_1$] ?

Example - Multi-objective





Strategy 1 (deterministic)

s_0 : east

s_1 : south

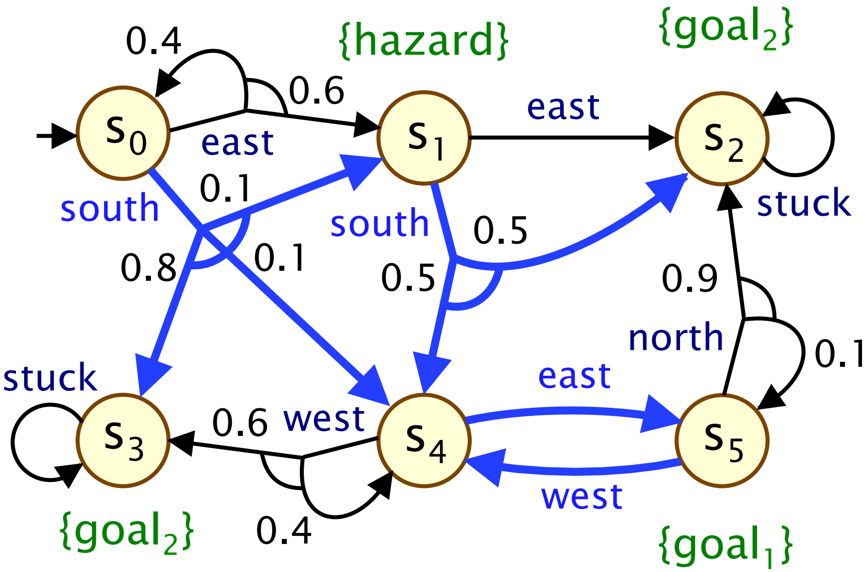
s_2 : –

s_3 : –

s_4 : east

s_5 : west

Example - Multi-objective



Strategy 2 (deterministic)

s_0 : south

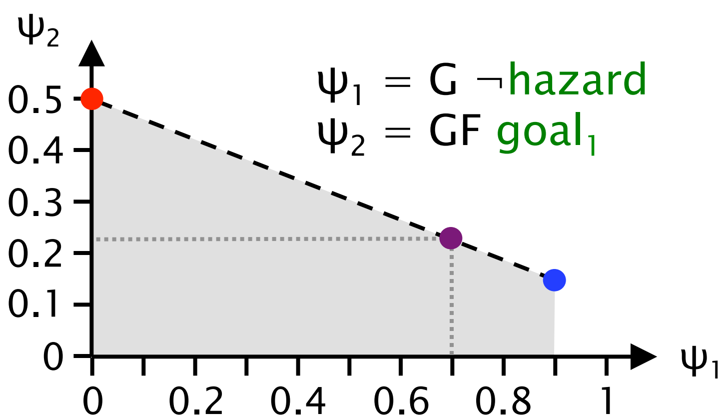
s_1 : south

s_2 : –

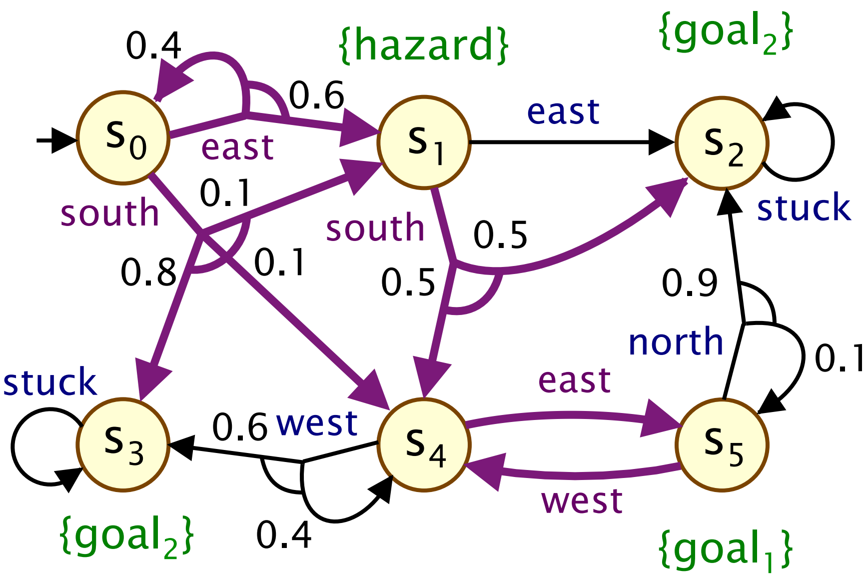
s_3 : –

s_4 : east

s_5 : west



Example - Multi-objective



Optimal strategy:
(randomised)

s_0 : 0.3226 : east

0.6774 : south

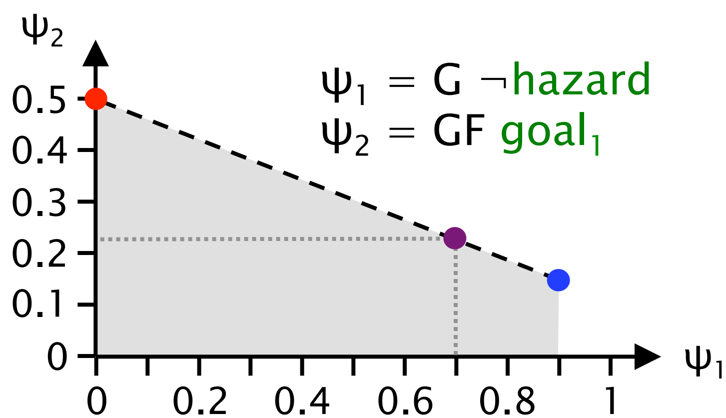
s_1 : 1.0 : south

s_2 : –

s_3 : –

s_4 : 1.0 : east

s_5 : 1.0 : west



Multi-objective: Applications

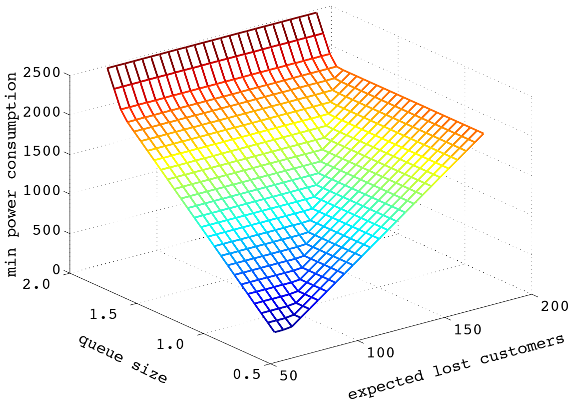
Synthesis of controllers for dynamic power management [TACAS'11]

IBM TravelStar VP disk drive

- switches between power modes:
- active/idle/idlelp/stby/sleep

MDP model in PRISM:

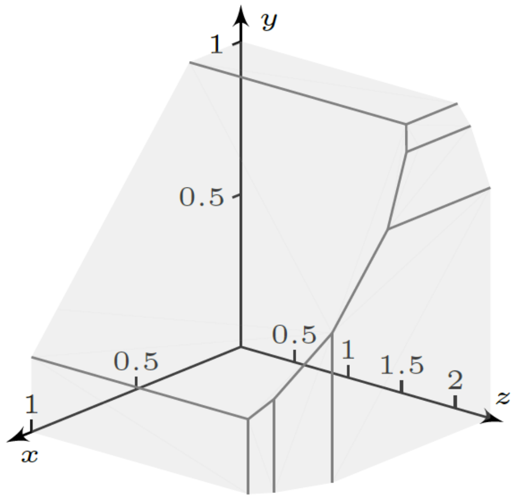
- power manager
- disk requests
- request queue
- power usage



Multi-objective:

"minimise energy consumption,
subject to constraints on:
(i) expected job queue size;
(ii) expected number of lost jobs

Synthesis of team formation strategies [CLIMA'11, ATVA'12]



Pareto curve:

x="probability of completing task 1";
y="probability of completing task 2";
z="expected size of successful team"

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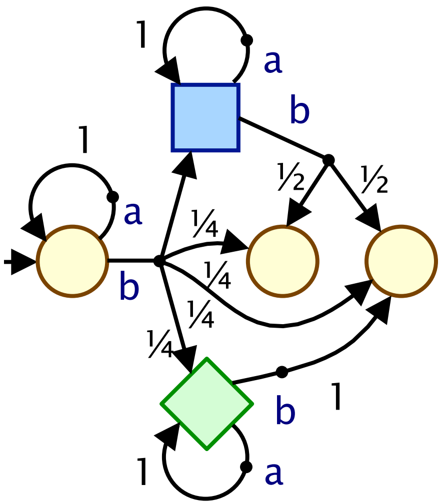
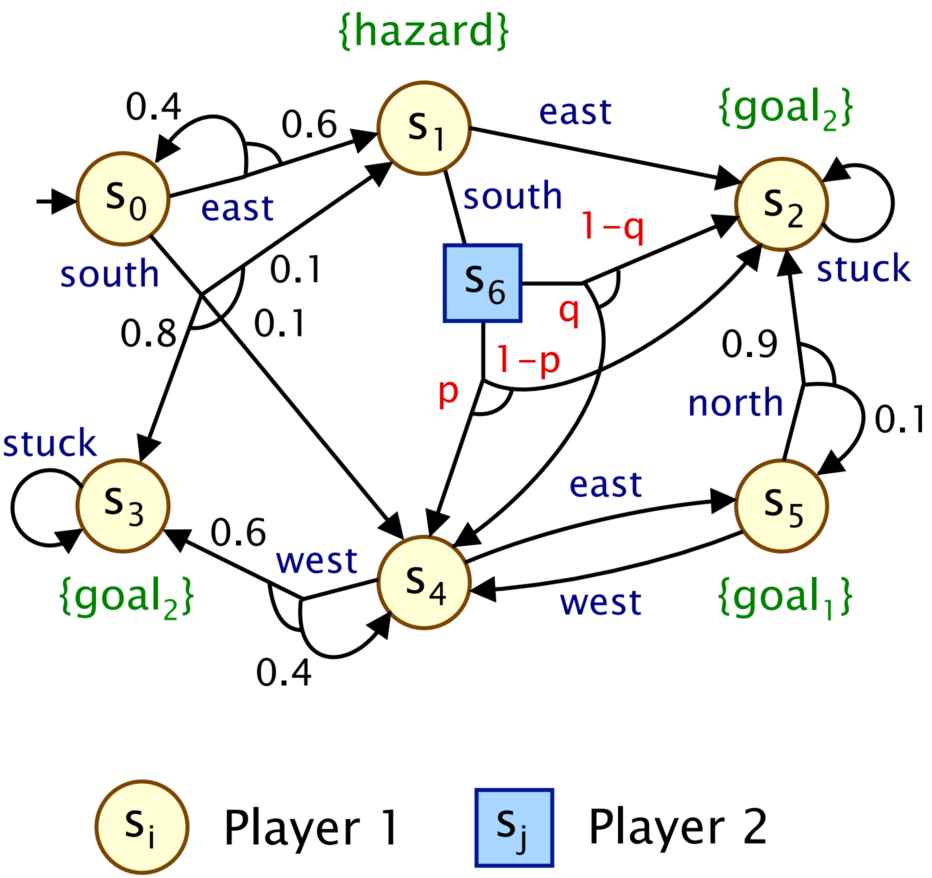
Stochastic multi-player games (SMGs)



- Stochastic multi-player games
  - players control states; choose actions
  - models competitive/collaborative behaviour
  - applications: security (system vs. attacker), controller synthesis (controller vs. environment), distributed algorithms/protocols, …
- Property specifications: rPATL
  - $\langle\langle\{1,2\}\rangle\rangle\ P_{\geq 0.95}\ [\ F^{\leq 45}\ \mathit{done}\ ]$: "can nodes 1,2 collaborate so that the probability of the protocol terminating within 45 seconds is at least 0.95, whatever nodes 3,4 do?"
  - formally: $\langle\langle C\rangle\rangle\psi$: do there exist strategies for players in C such that, for all strategies of other players, property ψ holds?
- Model checking [TACAS'12,FMSD'13]
  - zero sum properties: analysis reduces to 2-player games
  - PRISM-games: www.prismmodelchecker.org/games

Example – Stochastic games

- Two players: 1 (robot controller), 2 (environment)
  - probability of s_1–south→s_4 is in [p,q] = [0.5–Δ, 0.5+Δ]



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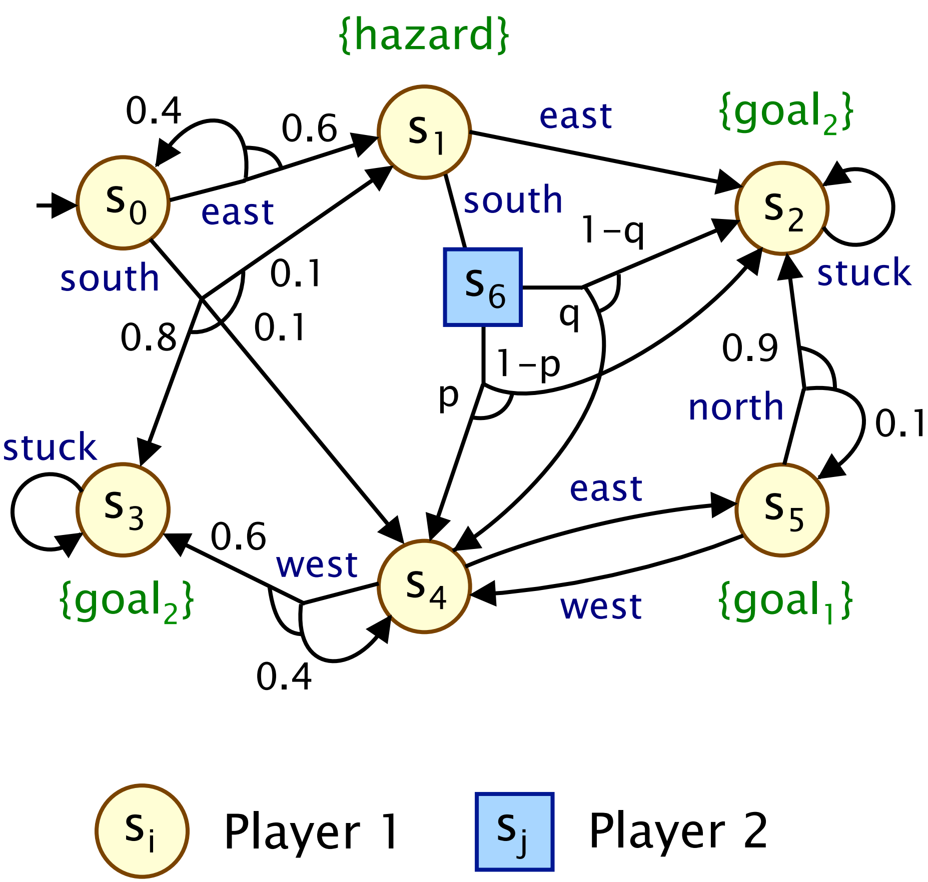

rPATL: $\langle\langle\{1\}\rangle\rangle\, P_{max=?}\,[\, F\, goal_1\,]$

Optimal strategies:
memoryless and deterministic

Computation: graph analysis
& numerical approximation

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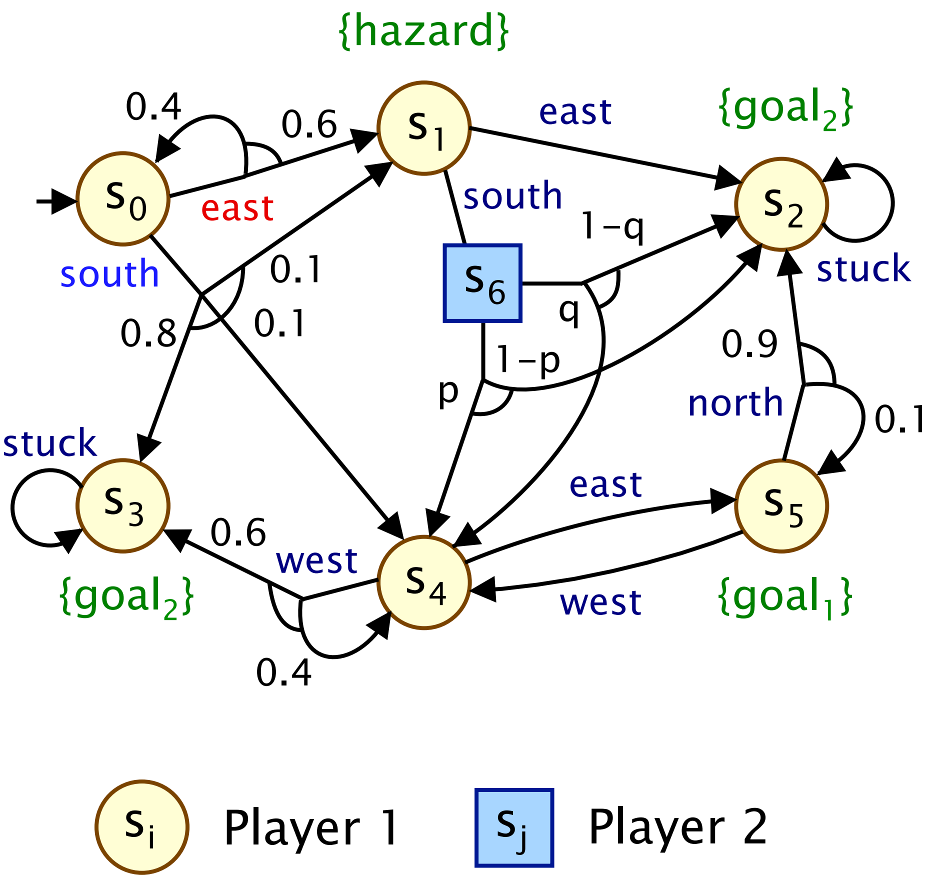

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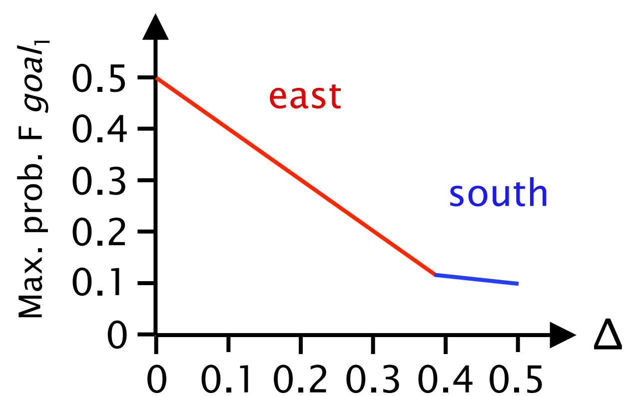
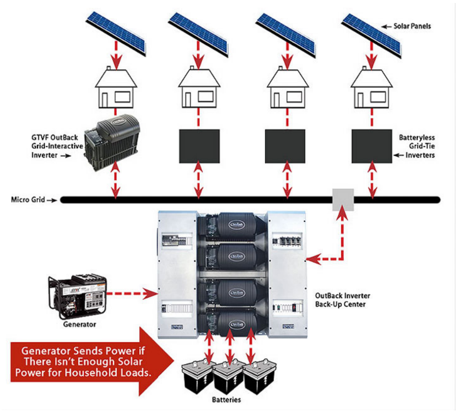


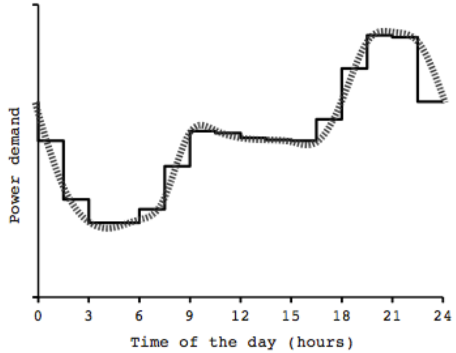
Application: Energy management

- Energy management protocol for Microgrid
  - randomised demand management protocol
  - random back-off when demand is high



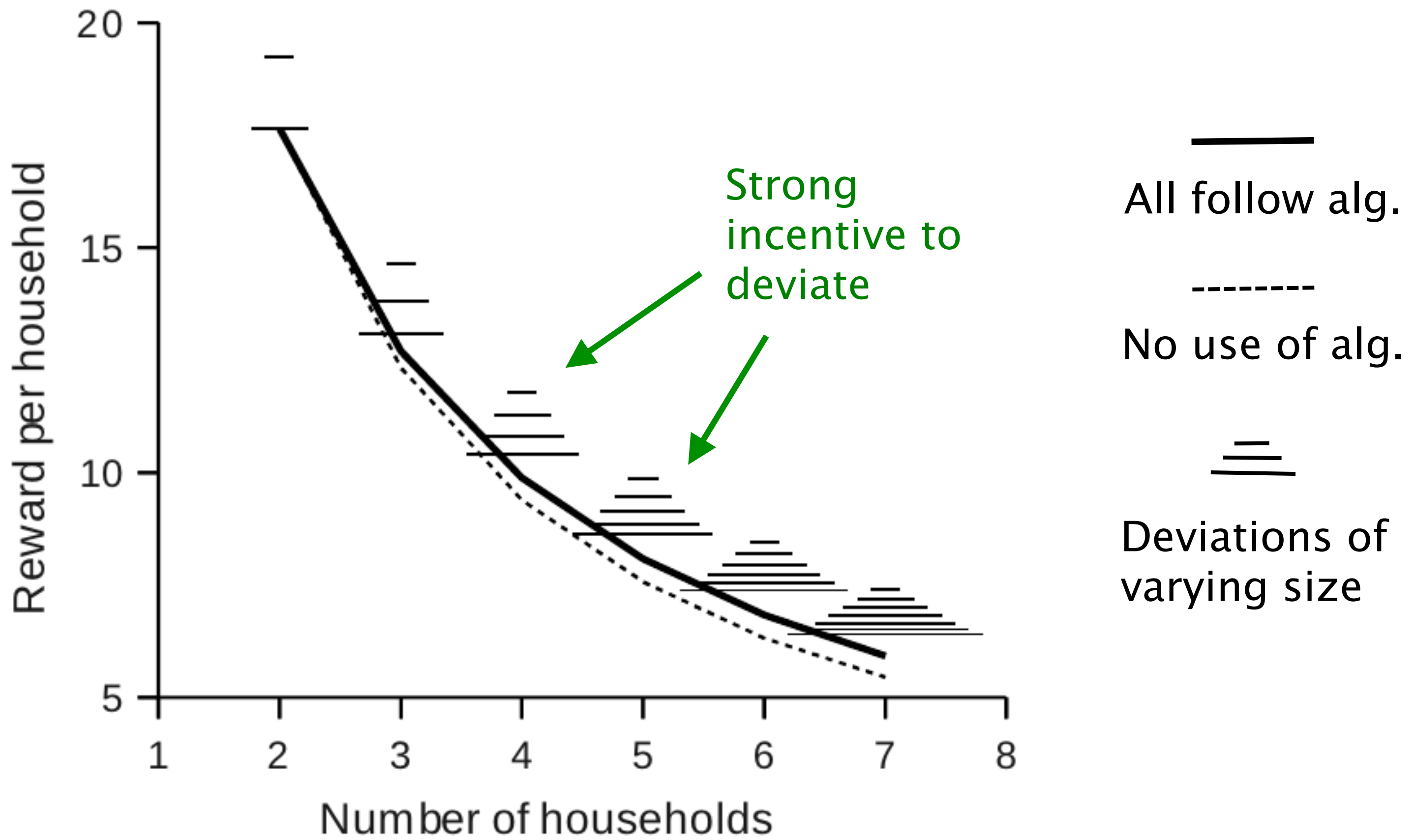
- Original analysis [Hildmann/Saffre'11]
  - protocol increases "value" for clients
  - simulation-based, clients are honest

- Our analysis
  - stochastic multi-player game model
  - clients can cheat (and cooperate)
  - model checking: PRISM-games
  - exposes protocol weakness (incentive for clients to act selfishly
  - propose/verify simple fix using penalties



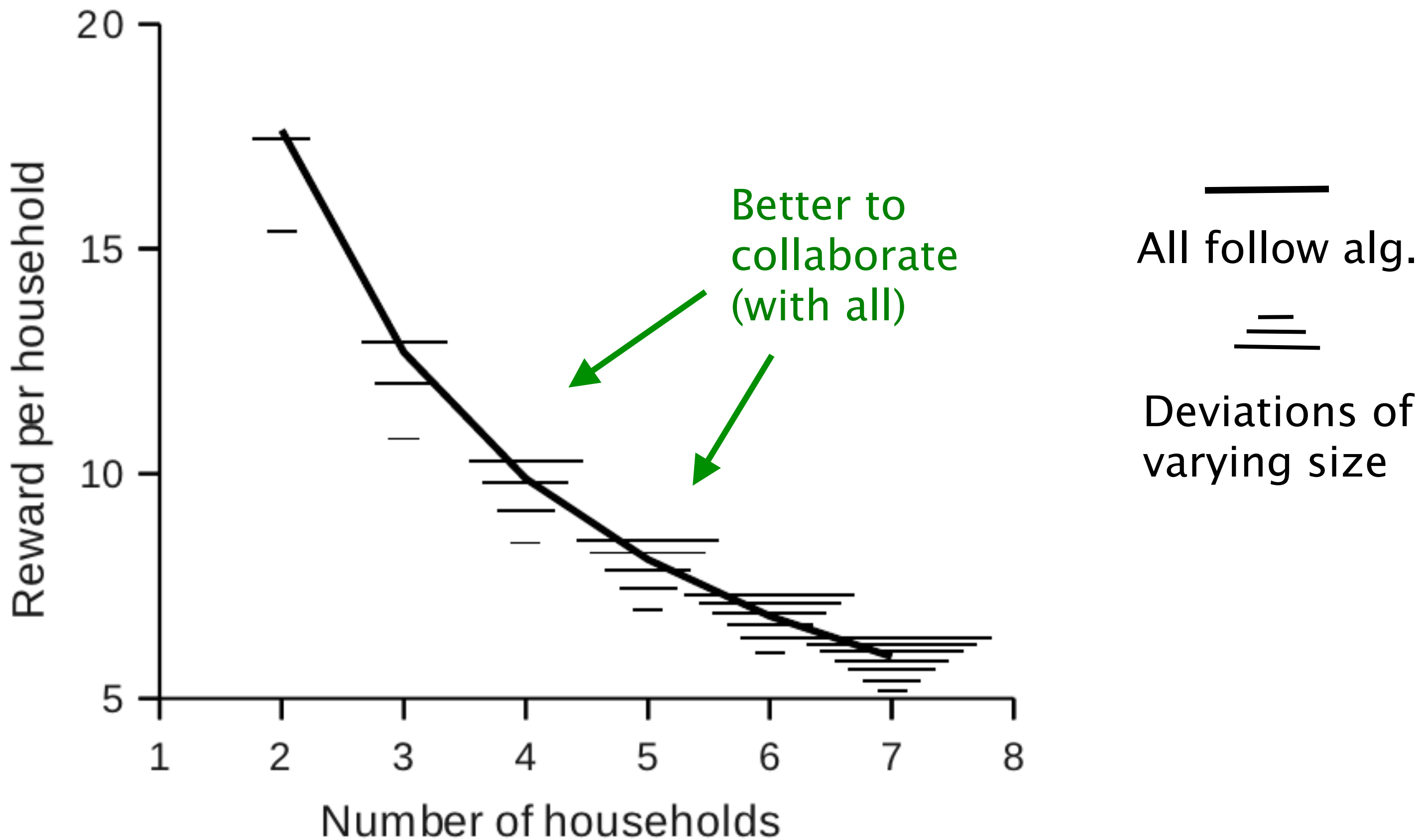
Results: Competitive behaviour

- Expected total value V per household
  - in rPATL: $\langle\langle C \rangle\rangle R^{r_C}_{max=?} [F^0\ time=max\ time]\ /\ |C|$
  - where r_C is combined rewards for coalition C



Results: Competitive behaviour

- Algorithm fix: simple punishment mechanism
  - distribution manager can cancel some loads exceeding c_{lim}



Conclusion

- Probabilistic model checking
  - verification vs. strategy synthesis
  - Markov decision processes, temporal logic, PRISM

- Recent directions and extensions
  - multi-objective probabilistic model checking
  - model checking for stochastic games

- Challenges
  - stochastic games: multi-objective, equilibria, richer logics
  - partial information/observability
  - probabilistic models with continuous time (or space)
  - scalability, e.g. symbolic methods, abstraction

www.prismmodelchecker.org