# Subsumption of concepts in $\mathcal{FL}_0$ for (cyclic) terminologies with respect to descriptive semantics is PSPACE-complete.

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# Description Logics

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- $\mathcal{A}$ – atomic concepts *(unary relations)*
- $\mathcal{R}$ – atomic roles *(binary relations)*

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# The reasoning tasks in DL

Knowledge base (or terminology) $\mathcal{T}$:

$$Human \doteq Mammal \sqcap \forall p^{arent}.Human$$

$$Elephant \doteq Mammal \sqcap \forall p^{arent}.Elephant$$

$$Adam \doteq Mammal \sqcap \forall p^{arent}.\bot$$

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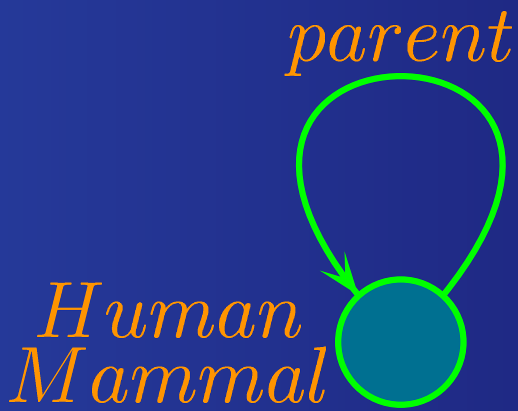


Should really all models satisfying $\mathcal{T}$ be considered?

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- $gfp$-semantics: cyclic definitions are evaluated in maximal possible way (*"all"-definitions*):

$$MOMO \doteq Man \sqcap \forall child.MOMO$$

# The small terminological language

| | | |
|---|---|---|
| $\mathcal{FL}_0$ ::= | $A$ \| | $A(x)$ |
| | $C_1 \sqcap C_2$ \| | $C_1(x) \wedge C_2(x)$ |
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| Subsumption in $\mathcal{FL}_0$ | Cyclic T-Boxes | Acyclic T-Boxes |
|---|---|---|
| *descriptive semantics* | in PSPACE, **PSPACE-hard** | co-NP-complete |
| *lfp-semantics* | PSPACE-complete | |
| *gfp-semantics* | PSPACE-complete | |

# The description graph

We focus our attention on terminologies $\mathcal{T}$ of the form:

$$A_i \doteq \forall R_{i,1}.B_{i,1} \sqcap \cdots \sqcap \forall R_{i,k_i}.B_{i,k_i} \qquad (1)$$

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The description graph $\mathcal{G}_{\mathcal{T}}$ is a graph, where:

- Nodes are labelled by concept names;
- Oriented edges are labelled by role names such that: the edge $e$ comes from the node $n_1$ to the node $n_2$ iff
  - $n_1$ is labelled by $A$, $n_2$ is labelled by $B$,
  - $e$ is labelled by $R$ and
  - $A \doteq \cdots \sqcap \forall R.B \sqcap \cdots \in \mathcal{T}$.

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Consider the terminology $\mathcal{T}$:

$$A \doteq \forall S.A \sqcap \forall T.B \sqcap \forall S.C$$

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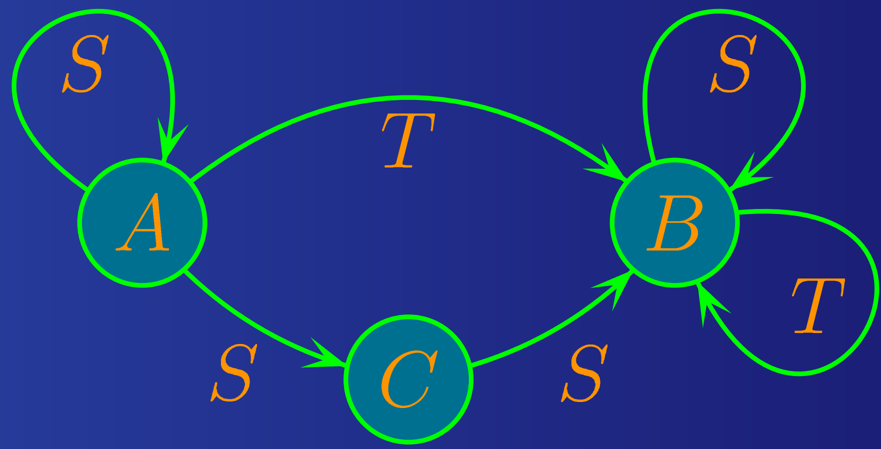

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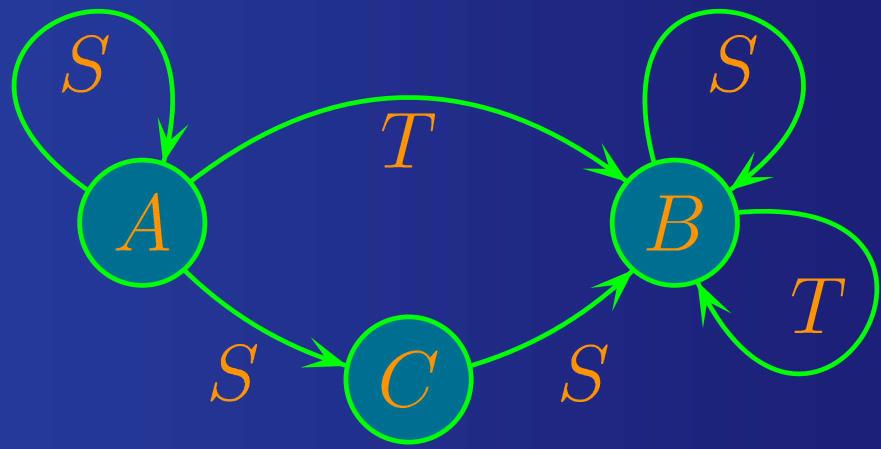


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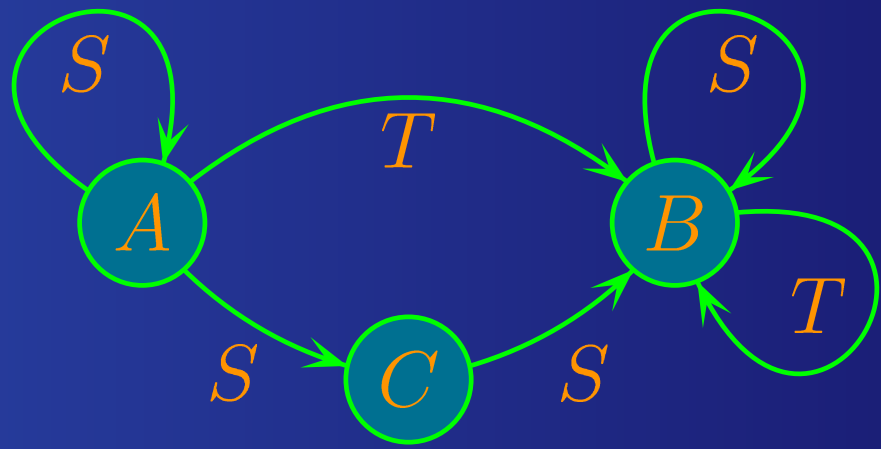


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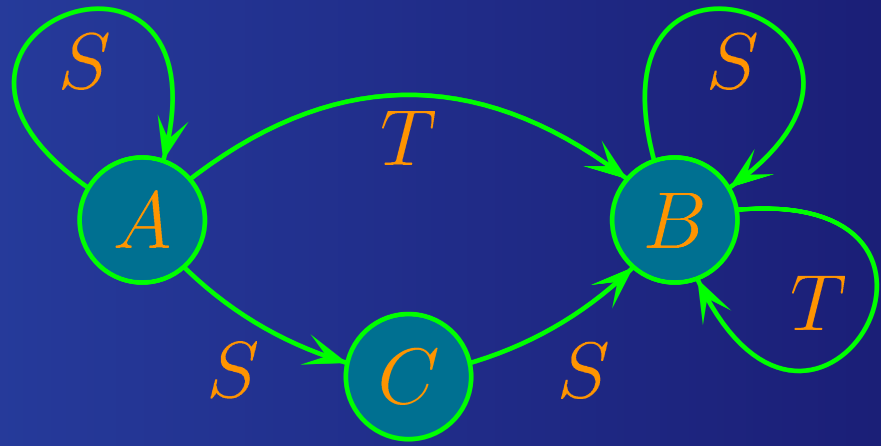


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- Repeating, we construct the infinite sequence of $a_i \in (\neg B)^{\mathcal{M}}$ with $(a_i, a_{i+1}) \in S^{\mathcal{M}}$ or $\in T^{\mathcal{M}}$.

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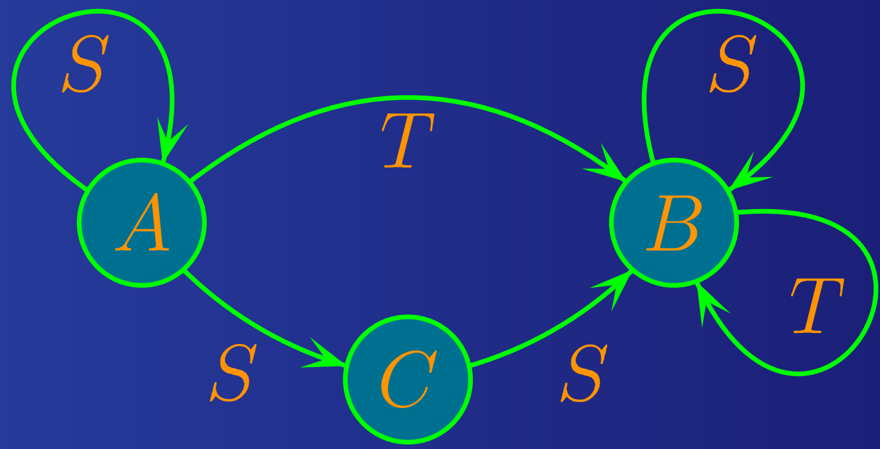


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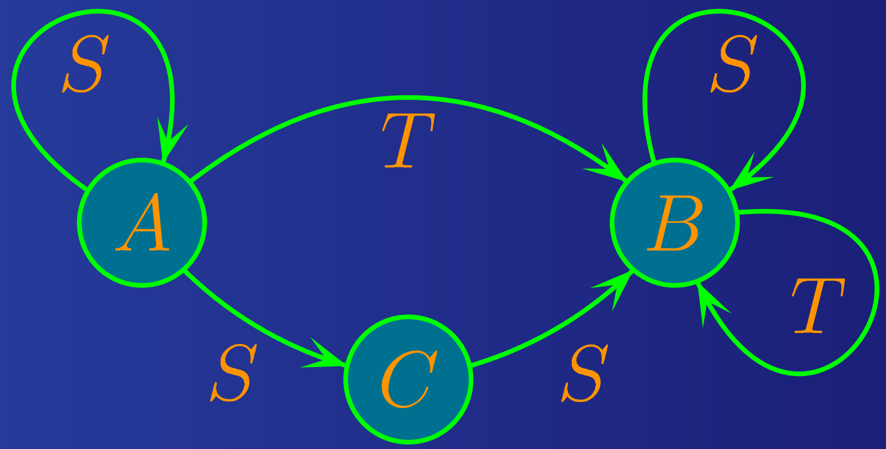


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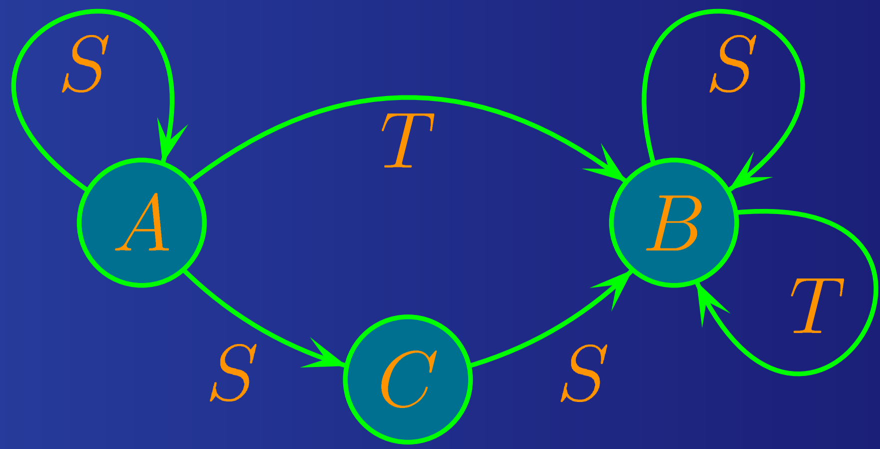


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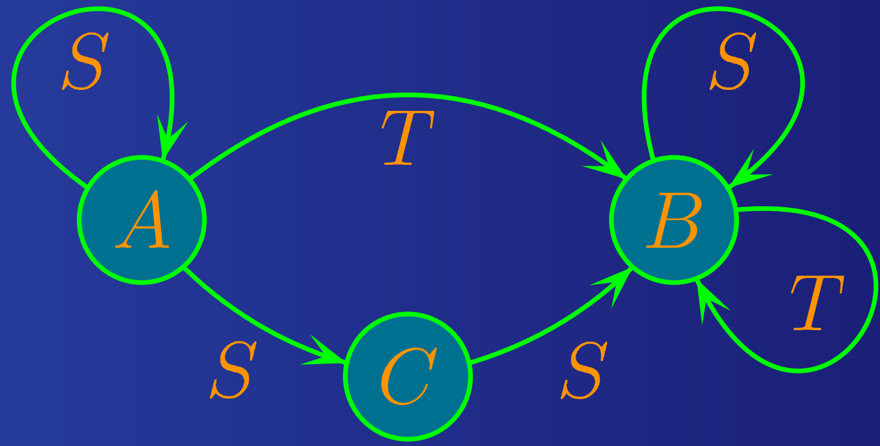


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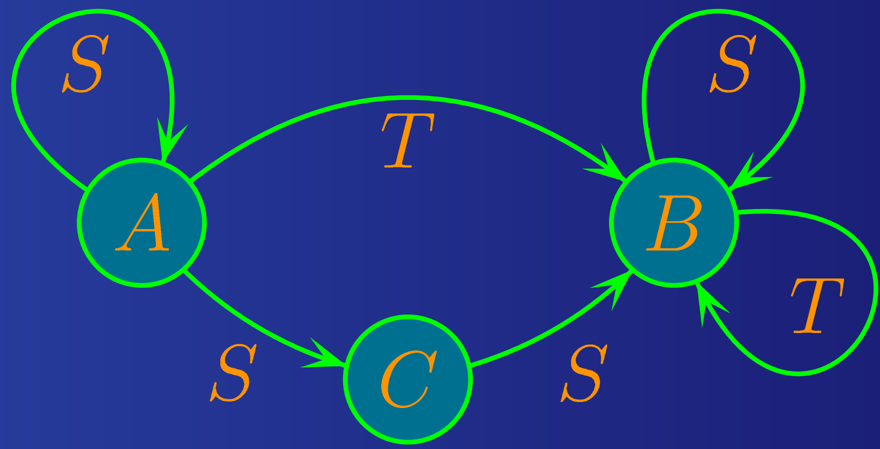


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- Since $a_i \in (\neg B)^{\mathcal{M}}$, no such $\mathcal{M}$ exists, thus $A \sqsubseteq_{\mathcal{T}} B$.

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The following can be shown using the similar arguments:

Lemma. (Characterization of concept subsumption)

$A \sqsubseteq_{\mathcal{T}} B$ iff in the description graph $\mathcal{G}_{\mathcal{T}}$
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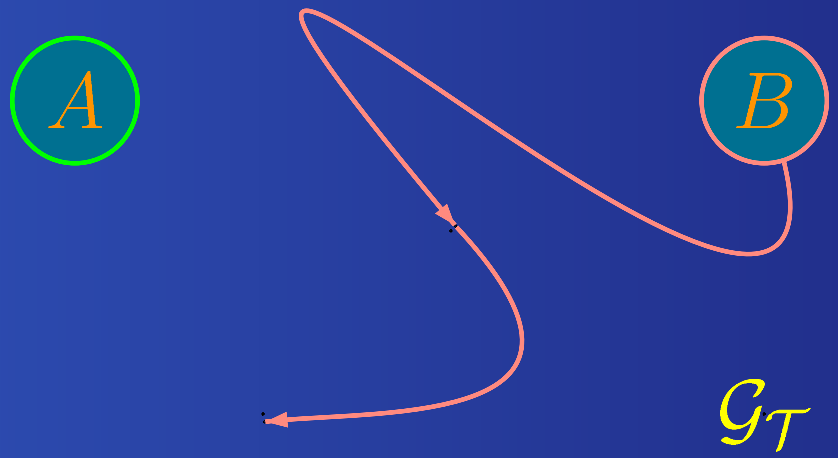

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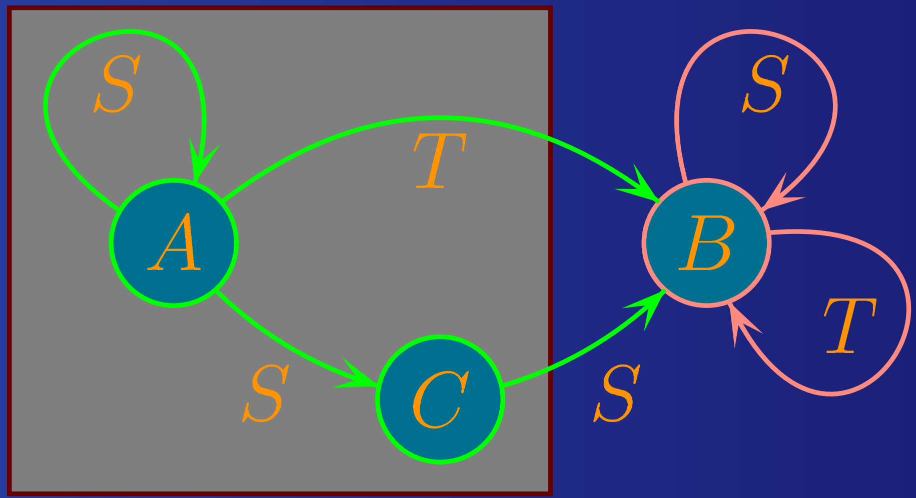

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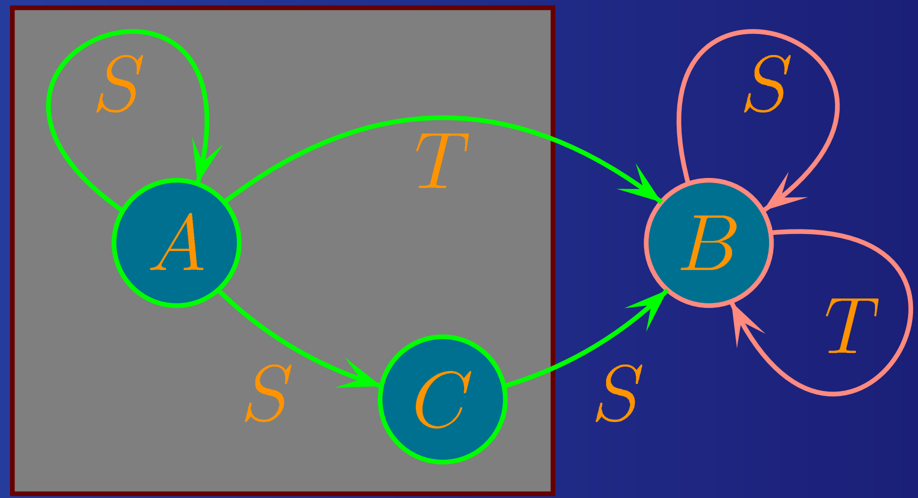


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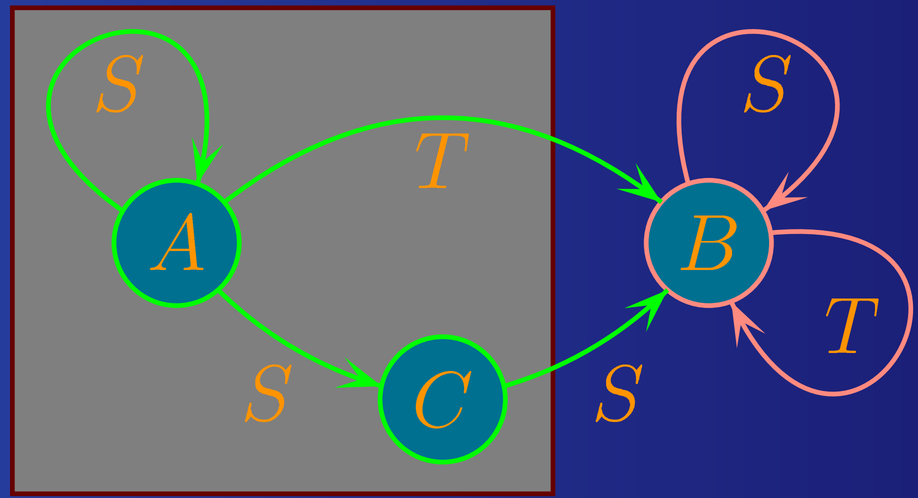


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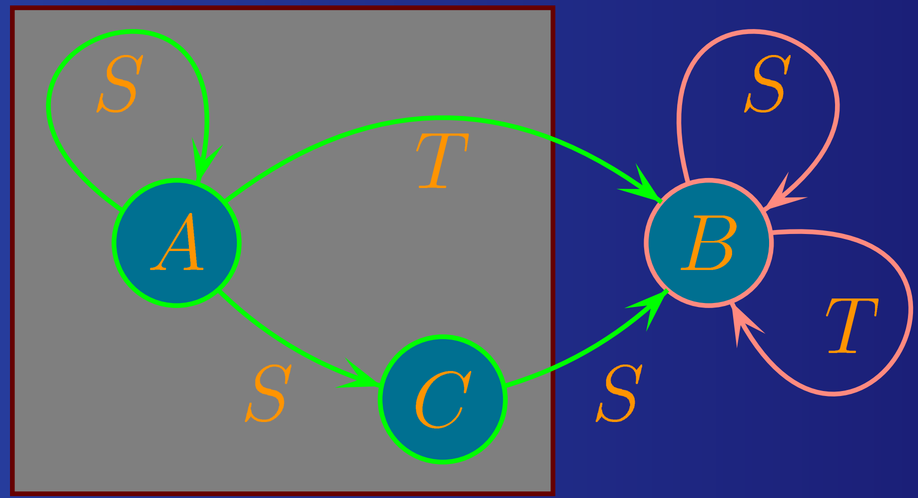


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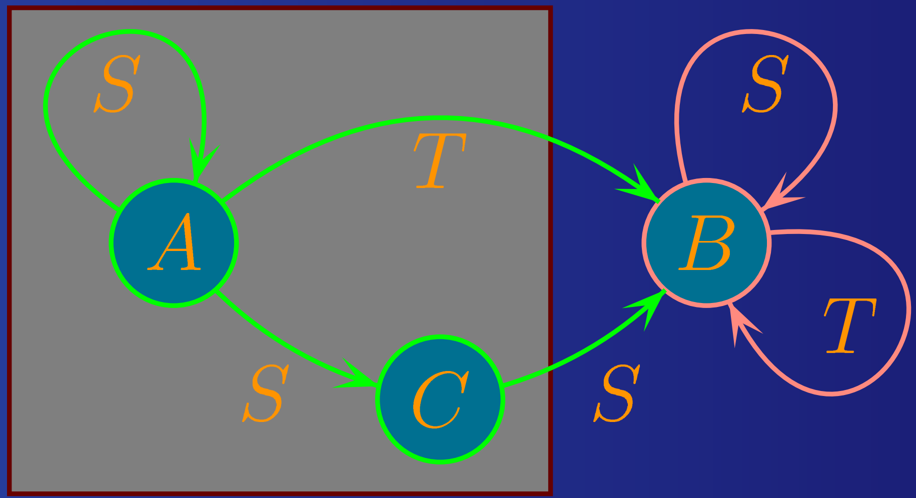


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- Thus $A \sqsubseteq_{\mathcal{T}} B$ and $C \not\sqsubseteq_{\mathcal{T}} B$

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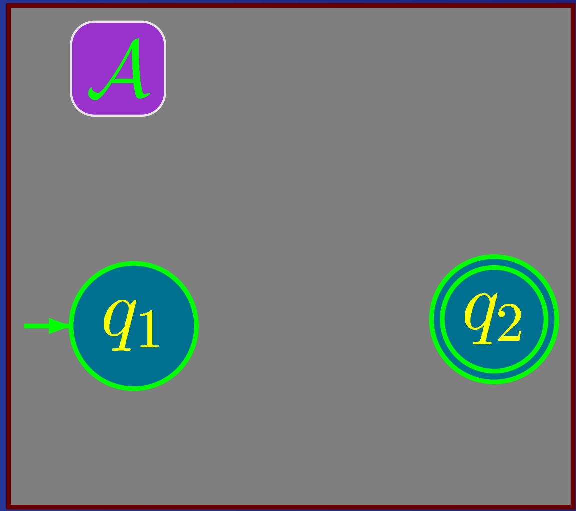

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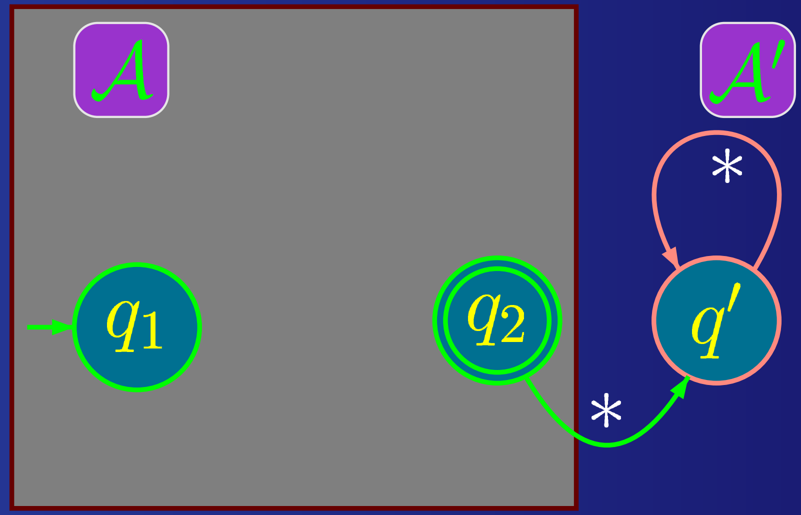


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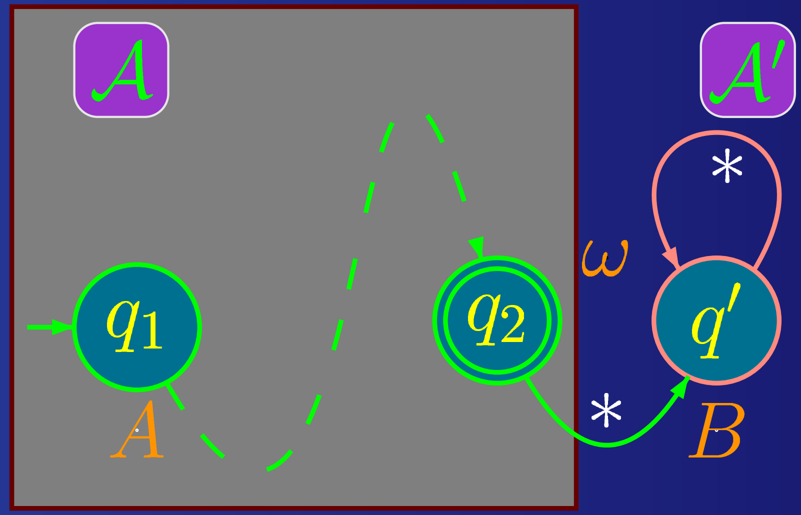


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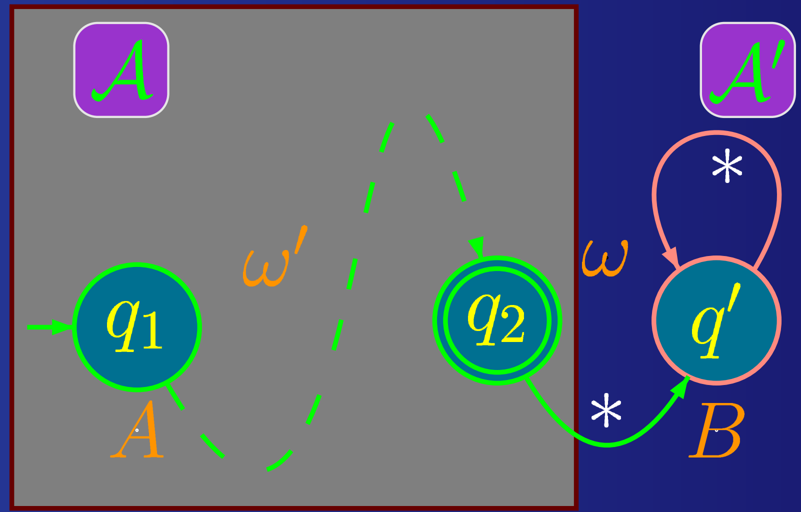


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- $A \sqsubseteq_{\mathcal{T}} B$ iff for any word $\omega \in \Sigma^\omega$ there is a finite prefix $\omega'$ which is accepted by $\mathcal{A}$.

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- Similar problem for finite automata and bu̎chi automata is known as the universality problem.

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- The alternative formulation for the problem:

$$L \cdot \Sigma^\omega = \Sigma^\omega \ ?$$

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**Lemma.(The reduction lemma)**

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**Theorem.**

*The universality problem for* $\mathbf{NFA}_p^\omega$ *is PSPACE-complete.*

**Corollary**

*Subsumption of concepts in* $\mathcal{FL}_0$ *for (cyclic) terminologies with respect to descriptive semantics is PSPACE-complete.*

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  Franz Baader (2002):
  Subsumption in $\mathcal{EL}$ is polynomial.
- Description logics with mixed semantics?
  T.Henzinger, O. Kupferman, R.Majumdar (2003):
  satisfiability of $\forall MC$ is PSPACE-complete,
  satisfiability of $\exists MC$ is NP-complete.
  However, implication problem ($\sim$ subsumption) is still EXPTIME.

# Thank you!

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