II. *Solution by Richard A. Vitale, Claremont Graduate School, Claremont, CA.* The desired point w^* is the limit of the sequence of shared vertices.

Let $v_0, v_1, \ldots, v_{n-1}$ be a clockwise enumeration of the vertices of P_0; extend this to an infinite periodic sequence $\{v_k\}$. Let $w_0 = v_0$ be the shared vertex of P_0 and P_1, and, generally, let w_k be the shared vertex of P_k and P_{k+1}. Then

$$w_{k+1} - w_k = f^{k+1}(v_{k+1} - v_k).$$

Solving this with the initial condition yields

$$w_N = f^N v_N + (1 - f) \sum_0^{N-1} f^k v_k,$$

which tends to $w^* = (1 - f)\Sigma_0^\infty f^k v_k$. The periodicity of the v sequence then gives

$$w^* = [(1 - f)/(1 - f^n)] \sum_0^{n-1} f^k v_k.$$

Also solved by J. C. Binz (Switzerland), J. Dou (Spain), R. B. Eggleton (Australia), W. Janous (Austria), L. Kuipers (Switzerland), R. E. Megginson, W. A. Newcomb, D. A. Rawsthorne, D. B. Shapiro, D. L. Shell, J. Ward and the proposer.

Shapiro pointed out that "the statement of the problem has some ambiguity, since it is not specified whether the n-gon P_{k+1} is to be traced in the same sense or in the opposite sense as P_k." The original intention was for P_{k+1} to be traced in the same sense as P_k.

ADVANCED PROBLEMS

Solutions of these Advanced Problems should be mailed in duplicate to Professor G. L. Alexanderson, Department of Mathematics, University of Santa Clara, Santa Clara, CA 95053, by November 30, 1984. The solver's full post-office address should be on each sheet.

6460. *Proposed by L. Richard Duffy (student), University of Chicago.*

It is well known that a discontinuous *linear* real function (i.e., one satisfying $f(x + y) = f(x) + f(y)$, all $x, y \in R$) must be unbounded on any open interval. Is there a linear function which is so pathological that it actually assumes *all* real values on any open interval?

6461. *Proposed by L. E. Mattics, University of South Alabama.*

Let p be a prime and suppose $1 \leqslant h < p$ where the order of h modulo p is $(p - 1)/\nu$. Prove that

$$(\nu - 1)\sqrt{p} + 1 \geqslant \left| \sum_{m=1}^{p-1} e^{2\pi i h^m/p} \right|.$$

6462. *Proposed by Randall J. LeVeque and Lloyd N. Trefethen, Courant Institute of Mathematical Sciences, New York University.*

Let S be the unit circle (or any other circle) in the complex plane. Let r be a rational function of type (n, n) with no poles on S.

(a) Show that $\|r'\|_1 \leqslant 4\pi n\|r\|_\infty$ where $\|\cdot\|_1$ and $\|\cdot\|_\infty$ are the L_1 and L_∞ norms on S. Note that $\|r'\|_1$ is the arclength of the image of S under r.

(b)* Show that $\|r'\|_1 \leqslant 2\pi n\|r\|_\infty$.