Modeling Ontologies Using OWL, Description Graphs and Rules

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1 Introduction

Ontologies often describe domains in which concepts are highly interconnected. An example is the description of *structured objects*, which consist of many parts connected in complex ways. Such objects abound, for example, in molecular biology and in the clinical sciences. Clinical ontologies such as GALEN, the Foundational Model of Anatomy (FMA) and the National Cancer Institute (NCI) Thesaurus are currently being used in large-scale applications, and they describe numerous structured objects. For example, FMA models the human hand as consisting of the fingers, the palm, various bones, blood vessels, and so on: all of which are highly interconnected.

The representation of structured objects or, in general, of domains involving highly interconnected concepts, poses well-known problems to OWL (and also to OWL 2). In fact, OWL can only faithfully describe domains where the objects are connected in a certain tree-like manner since OWL enjoys (a variant of) the tree model property, which is responsible for the decidability of reasoning. This property, however, also prevents sufficiently accurate description of complex domains since OWL axioms cannot describe arbitrary relational structures.

To address this lack of expressivity we propose to extend OWL with *description graphs*, which consist of vertices labeled with concepts and edges labeled with roles. These graphs are schema-level statements that specify patterns of connections between objects. In addition, we allow for first-order rules [1] to represent conditional statements about graphs. For example, we can represent the hand and its parts using description graphs and use rules to represent statements such as 'if a bone in the hand is fractured, then the hand is fractured as well'. We can then use OWL to model non-structural aspects of the domain, such the fact that a medical doctor is a person with an MD degree.

We thus obtain a powerful knowledge representation formalism that addresses our limitations in expressive power, but it is undecidable. It is widely recognized, however, that reasoning algorithms are more likely to be effective in practice if the underlying logics are decidable. We have identified the main causes for undecidability and investigated restrictions under which our formalism can be made decidable while still solving most of the identified expressivity problems.

To this end, we have exploited a fundamental observation about the representation of structured objects, namely that modeling them requires only a bounded

(but possibly large) number of objects. The fact that the domain is bounded is intrinsic to the modeling problem since structured objects are always modeled up to a certain level of granularity. For example, a human body consists of a certain number of organs. These organs can be decomposed into smaller parts; however, each such decomposition will eventually reach the parts that the modeler does not want to describe, or know how to describe any further. For example FMA describes the skeleton of the hand, but it does not describe the inner structure of the distal phalanges of the fingers. The level of granularity of the representation naturally determines a bound in the sequence of description graphs one needs to represent and in the size of each of those graphs.

Effectively, when describing the structure of an object, we often obtain a hierarchy of parts. This hierarchy does not contain cycles; for example, if the carpal bones are part of the hand, then it will most likely not be the case that the hand will eventually become a part of the carpal bones. To reflect the acyclic nature of the representation, we impose an acyclicity condition in our formalism which ensures that the description graphs representing each of those parts are arranged in a hierarchical manner and lead to a bounded representation.

Our acyclicity condition, however, is not sufficient to ensure decidability due to the arbitrary interaction between OWL axioms and rules [2]. To attain decidability, we limit their interaction by introducing role separation, which places restrictions in the usage of atomic roles in the OWL axioms, graphs and rules.

This paper summarizes the results published in a number of recent papers [3, 4]. Here, we focus on presenting our formalism by means of examples and on pointing out future applications. We refer the readers interested in the details of our more technical results to [3, 4]. We shall assume, however, that the reader is familiar with OWL and the basics of description logics (DLs).

2 Problems with Modeling Complex Structures

To understand the limitations of modeling structured objects in DLs (and hence in OWL), we consider the problem of modeling the skeleton of the human hand (see Figure 1a). The carpal bones form the base of the hand. The central part contains the metacarpal bones, one leading to each finger. The fingers consist of phalanges: the proximal phalanges are connected to the metacarpal bones, and all fingers apart from the thumb contain a middle phalanx between the proximal and the distal phalanx. This structure can be conceptualized as in Figures 1b–1e.

Figures 1b–1e could be represented in DLs using an ABox $\mathcal{A}$. ABox assertions, however, represent concrete data; thus, $\mathcal{A}$ would represent the structure of *one particular* hand. In this paper, we are concerned with modeling structured objects *at the schema level*—that is, we want to describe the general structure of *all* hands. We should be able to instantiate such a description many times. For example, if we say that each patient has a hand, then, for each concrete patient, we should instantiate a *different* hand, each of the structure shown in Figures 1b–1e. This clearly cannot be achieved using ABox assertions.

We can give a logical, schema-level interpretation to Figures 1b–1e by treating vertices as concepts and arrows as *participation constraints* specifying their relationships. For example, *Hand* and *Index_finger* are concepts and the arrow between them says that the index is a part of the hand. Participation constraints are represented in ontologies using DL axioms such as (1)–(5)[3].

Let $\mathcal{K}$ be a DL knowledge base containing the following axioms.[4]

$$\text{(1)} \quad \mathit{Index\_finger} \sqsubseteq \exists \mathit{hasPart}.\mathit{Distal\_phalanx}$$

$$\text{(2)} \quad \mathit{Index\_finger} \sqsubseteq \exists \mathit{hasPart}.\mathit{Middle\_phalanx}$$

$$\text{(3)} \quad \mathit{Proximal\_phalanx} \sqsubseteq \exists \mathit{isPartOf}.\mathit{Index\_finger}$$

$$\text{(4)} \quad \mathit{Distal\_phalanx} \sqsubseteq \exists \mathit{attachedTo}.\mathit{Middle\_phalanx}$$

$$\text{(5)} \quad \mathit{Middle\_phalanx} \sqsubseteq \exists \mathit{attachedTo}.\mathit{Proximal\_phalanx}$$

$$\text{(6)} \quad \mathsf{Sym}(\mathit{attachedTo})$$

$$\text{(7)} \quad \mathit{isPartOf} \equiv \mathsf{Inv}(\mathit{hasPart})$$

Let I be an interpretation corresponding to Figure 1e in the obvious way. Clearly, I satisfies $\mathcal{K}$, which justifies the formalization of Figure 1e using $\mathcal{K}$.

Unfortunately, the ontology $\mathcal{K}$ is underconstrained: some models of $\mathcal{K}$ do not correspond to the actual structure of the index finger from Figure 1e. Axioms (2) and (4) imply the existence of two middle phalanges of the index finger, but $\mathcal{K}$ does not state that these two middle phalanges must be the same object. Thus, an interpretation I' corresponding to Figure 2 is also a model of $\mathcal{K}$.

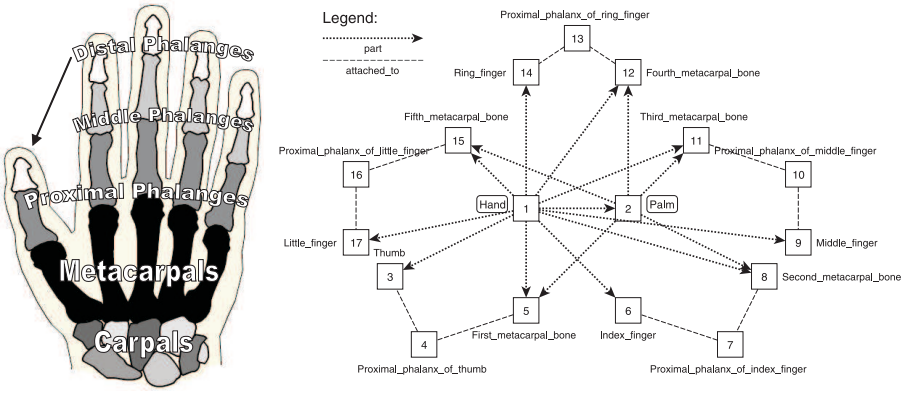
This discrepancy prevents us from drawing any conclusions that rely on the nontree connections in the structure; for example, if the index finger has a broken distal phalanx, then we should conclude that the phalanx adjacent to the middle phalanx is broken (since this is the same broken phalanx). Furthermore, it can also cause problems with the performance of reasoning. For example, we might use axioms (2)–(7) to describe the relationships between the index finger, its proximal phalanx and its middle phalanx.

While admitting a model corresponding to Figure 1e, these axioms do not state that the index finger in (3) is a part of the "initial" index finger. Hence, the interpretation I'' from Figure 3 is also a model of these axioms.

In fact, the latter model is 'canonical' in the sense that it reflects the least amount of information derivable from the axioms. In order to disprove an entailment, a DL reasoner will try to construct such 'canonical' model. In practice, these models can be highly repetitive and much larger than the intended ones, which, according to our experience, is the main reason why DL reasoners cannot process ontologies such as FMA and certain versions of GALEN.

[3] The role *attached_to* is symmetric, so we do not orient the edges labeled with it.

[4] For brevity, we use in these examples the terms Distal_phalanx, Middle_phalanx and Proximal_phalanx as an abbreviation for Distal_phalanx_of_index_finger, Middle_phalanx_of_index_finger and Proximal_phalanx_of_index_finger respectively. It should be clear from the context that we are referring to the index finger.



(a) Anatomy of the Hand (b) Hand (G_{hand})

(c) Finger (G_{finger}) (d) Thumb (G_{thumb}) (e) Index (G_{index_finger})

Fig. 1: The Anatomy of the Hand and its Conceptual Models

We propose to extend $\mathcal{K}$ with additional axioms that make *all* its models correspond as much as possible to the intended conceptualization in Figures 1b–1e. Such axioms, however, cannot be stated in DLs since DL languages exhibit (a variant of) the *tree model property* [5]: whenever a DL knowledge base $\mathcal{K}$ has a model, it has a model of a certain tree shape. The relationship between the index finger and its phalanges cannot be represented as a tree. Hence, in order to faithfully represent Figures 1b–1e, we must leave the confines of DLs and OWL.

3 The Formalism

We now present our formalism. We first introduce description graphs.

Definition 1 (Description Graph). *An ℓ-ary* description graph*is a directed labeled graph $G = (V, E, \lambda, M)$ with $V = \{1, \ldots, \ell\}$ a set of* vertices*, $E \subseteq V \times V$ a set of* edges*, and λ a labeling function that assigns a set of atomic concepts or the negation of atomic concepts $\lambda\langle i\rangle$ to each vertex $i \in V$ and a set of atomic roles $\lambda\langle i, j\rangle \subseteq N_R$ to each edge $\langle i, j\rangle \in E$. Finally, $M \subseteq N_C$ is a set of* main concepts *for G. For A an atomic concept, V_A is the set of vertices that contain A in their label; that is, $V_A = \{k \in V \mid A \in \lambda\langle k\rangle\}$.*

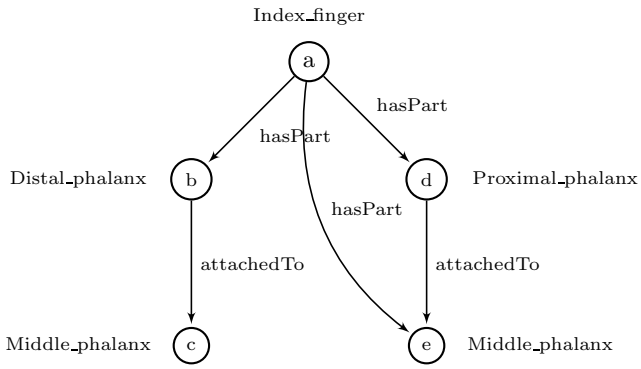


Fig. 2: Unintended Model I'

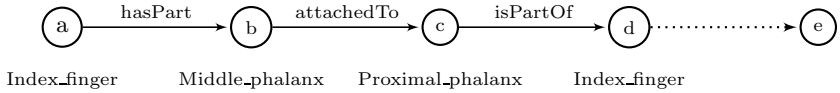


Fig. 3: Unintended Infinite Model I''

Thus, description graphs are labeled graphs where the nodes are labeled with concepts and the edges with roles. The main concepts indicate the objects whose structure is defined by the graphs. For example, the main concepts for the graph in Figure 1b (framed with rounded rectangles) are *Hand* and *Palm*, meaning that this graph defines the structure of the hand and the palm. Intuitively, an instance of a main concept implies the existence of a graph instance.

Definition 2 (Rules). *Let N_I and N_V be disjoint sets of individuals and variables. An* atom *is of the form $C(s)$, $R(s,t)$, or $s \approx t$, for $s,t \in N_I \cup N_V$, C a concept, and R a role. A* rule *is an expression of the form*

$$U_1 \wedge \ldots \wedge U_m \rightarrow V_1 \vee \ldots \vee V_n \tag{8}$$

where U_i and V_j are atoms, $m \geq 0$, and $n \geq 0$. W.l.o.g we assume that the body never contains $\approx$. The conjunction $U_1 \wedge \ldots \wedge U_m$ is called the body*, and the disjunction $V_1 \vee \ldots \vee V_n$ is called the* head*. Variables x and y are* directly connected *in a rule r if they both occur in a body atom of r, and* connected *is the transitive closure of directly connected. A rule r is* connected *if each pair of variables x and y occurring in r is connected in r.*

A graph rule *is a rule of the form* (8) *where all concepts and roles in atoms are atomic, and that can also contain* graph atoms *of the form $G(t_1, \ldots, t_k)$, for G an ℓ-ary description graph and $t_i \in N_I \cup N_V$.*

Next, we introduce graph specializations, which allow us to represent objects at different levels of abstraction. For example, we would like to describe the

abstract structure common to all fingers as shown in Figure 1c; then, we should be able to specialize this structure for the index finger and introduce the middle phalanx, as in Figure 1e. The graph specialization $G_{finger} \lhd G_{thumb}$ states that the graph for the thumb specializes the graph for a finger.

Definition 3 (Graph Specialization). *A* graph specialization *is an axiom of the form $G_1 \lhd G_2$, where $G_1 = (V_1, E_1, \lambda_1, M_1)$ and $G_2 = (V_2, E_2, \lambda_2, M_2)$ are description graphs with $V_1 \subseteq V_2$.*

Next, we introduce axioms that allow us to properly connect graph instances. For example, G_{hand} contains the vertices 3 and 4 for the thumb and its proximal phalanx, which correspond to the vertices 1 and 3 of G_{thumb}. We can specify this correspondence using a *graph alignment* of the form $G_{hand}[3,4] \leftrightarrow G_{thumb}[1,3]$. Intuitively, this ensures that it is not possible for G_{hand} and G_{thumb} to share the thumb without sharing the proximal phalanx as well.

Definition 4 (Graph Alignment). *A* graph alignment *is an expression of the form $G_1[v_1, \ldots, v_n] \leftrightarrow G_2[w_1, \ldots w_n]$, where G_1 and G_2 are description graphs with sets of vertices V_1 and V_2, respectively, $v_i \in V_1$ and $w_i \in V_2$ for $1 \leq i \leq n$.*

Finally, we define GBoxes and graph-extended KBs.

Definition 5 (Formalism). *A* graph box *(GBox) is a tuple $\mathcal{G} = (\mathcal{G}_G, \mathcal{G}_S, \mathcal{G}_A)$ where $\mathcal{G}_G$, $\mathcal{G}_S$, and $\mathcal{G}_A$ are finite sets of description graphs, graph specializations over $\mathcal{G}_G$, and graph alignments over $\mathcal{G}_G$. ABoxes are extended to allow for* graph assertions *of the form $G(a_1, \ldots, a_\ell)$ for G an ℓ-ary graph. A* graph-extended knowledge base *is a 4-tuple $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ where $\mathcal{T}$ is a TBox, $\mathcal{P}$ is a program with a finite number of connected rules, $\mathcal{G}$ is a GBox, and $\mathcal{A}$ is an ABox.*

Next, we define the semantics of the formalism.

Definition 6 (Semantics). *An interpretation $I = (\triangle^I, \cdot^I)$ is defined as usual, and it interprets each ℓ-ary description graph G as an ℓ-ary relation over $\triangle^I$; that is, $G^I \subseteq (\triangle^I)^\ell$. A graph assertion is satisfied in I, written $I \models G(a_1, \ldots, a_\ell)$, iff $\langle a_1^I, \ldots, a_\ell^I \rangle \in G^I$. Satisfaction of a description graph, graph specialization, and graph alignment is defined in Table 1. Satisfaction of $\mathcal{T}$, $\mathcal{P}$ and $\mathcal{A}$ is standard. A knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ is satisfied in I, written $I \models \mathcal{K}$, if all its components are satisfied in I.*

Thus, each ℓ-ary graph G is interpreted as an ℓ-ary relation G^I in which each tuple corresponds to an instance of G in the interpretation. The key and disjointness properties in Table 1 ensure that no two distinct instances of G can share a vertex; for example, no two distinct instances of G_{hand} can share the vertex for the thumb. These properties prevent, for example, the occurrence of infinite 'chains' of G_{hand} and therefore are needed to ensure that the representation of the structured objects is bounded. The start property in Table 1 ensures that each instance of a main concept A of G occurs in an instance of G. For example, since *Hand* is a main concept for G_{hand}, each instance of *Hand* must occur as vertex 1 in an instance of G_{hand}.

Table 1: Satisfaction of GBox Elements in an Interpretation

$I \models G$ for $G = (V, E, \lambda, M)$ iff
Key property:
$\forall x_1, \ldots, x_\ell, y_1, \ldots, y_\ell \in \triangle^I$:
$\langle x_1, \ldots, x_\ell \rangle \in G^I \wedge \langle y_1, \ldots, y_\ell \rangle \in G^I \wedge \bigvee_{1 \leq i \leq \ell} x_i = y_i \rightarrow \bigwedge_{1 \leq j \leq \ell} x_j = y_j$
Disjointness property:
$\forall x_1, \ldots, x_\ell, y_1, \ldots, y_\ell \in \triangle^I : \langle x_1, \ldots, x_\ell \rangle \in G^I \wedge \langle y_1, \ldots, y_\ell \rangle \in G^I \rightarrow \bigwedge_{1 \leq i < j \leq \ell} x_i \neq y_j$
Start property: for each atomic concept $A \in M$,
$\forall x \in \triangle^I : x \in A^I \rightarrow \exists x_1, \ldots, x_\ell \in \triangle^I : \langle x_1, \ldots, x_\ell \rangle \in G^I \wedge \bigvee_{k \in V_A} x = x_k$
Layout property:
$\forall x_1, \ldots, x_\ell \in \triangle^I : \langle x_1, \ldots, x_\ell \rangle \in G^I \rightarrow$ $\bigwedge_{i \in V,\ B \in \lambda\langle i \rangle} x_i \in B^I \wedge \bigwedge_{\langle i,j \rangle \in E,\ R \in \lambda\langle i,j \rangle} \langle x_i, x_j \rangle \in R^I$
$I \models G_1 \lhd G_2$ iff
$\forall x_1, \ldots, x_{\ell_2} \in \triangle^I : \langle x_1, \ldots, x_{\ell_1}, \ldots, x_{\ell_2} \rangle \in G_2^I \rightarrow \langle x_1, \ldots, x_{\ell_1} \rangle \in G_1^I$
$I \models G_1[v_1, \ldots, v_n] \leftrightarrow G_2[w_1, \ldots w_n]$ iff, for each $1 \leq i \leq n$,
$\forall x_1, \ldots, x_{\ell_1}, y_1, \ldots, x_{\ell_2} \in \triangle^I$:
$\langle x_1, \ldots, x_{\ell_1} \rangle \in G_1^I \wedge \langle y_1, \ldots, y_{\ell_2} \rangle \in G_2^I \wedge x_{v_i} = y_{w_i} \rightarrow \bigwedge_{1 \leq j \leq n} x_{v_j} = y_{w_j}$
Note: $\ell_{(i)}$ is the arity of the description graph $G_{(i)}$.

Graph specializations are interpreted as inclusions over the graph relations; for example, $G_{finger} \lhd G_{index\_finger}$ means that each instance of an index finger is also an instance of a finger. The two graphs share all the vertices of the more general graph, and the more specific graph can introduce additional vertices. Finally, graph alignments state that, whenever two graphs share some vertex from the specified list, then they share all other vertices from the list as well. For example, the alignment $G_{hand}[3, 4] \leftrightarrow G_{thumb}[1, 3]$ states that, if instances of G_{hand} and G_{thumb} share vertices 3 and 1, respectively, then they must also share vertices 4 and 3, respectively.

The main reasoning problem is satisfiability checking, as subsumption and instance checking can be reduced to satisfiability as usual.

4 Other Applications

The potential applications of our formalism are not restricted to anatomy. In particular, our formalism could be applied to domains where the concepts used in the representation are highly interconnected, but where the number of arbitrarily interconnected objects has a natural bound in size. In this section, we provide a

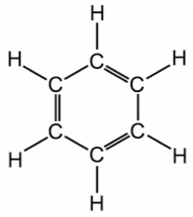


Fig. 4: The Benzene Ring

few additional examples of domains that cannot be faithfully represented using OWL, but which could be modeled using our formalism.

Chemical Compounds: The precise description of molecules is an important issue in bio-informatics [6]. In particular, a suitable formal representation of molecules and chemical compounds would enable the integration of chemical information at different levels of granularity [6]. The description of molecules involves objects whose structure is not tree-like. For example, a benzene is an organic chemical compound which naturally occurs in crude oil. Benzenes are hydrocarbons— compounds whose structure involves at least one benzene ring (see Figure 4). Apart from the non-tree benzene ring, hydrocarbons could contain arbitrary chains of carbon and hydrogen atoms, whose structure is tree-like.

Scientific Workflows: Scientific workflows are depictions of a sequence of operations, such as actions involved in scientific experiments. Scientific workflows are often represented as DAGs with the nodes being computational components and the edges paths along which data and results can flow between components. The precise description of workflows is increasingly important, for example, in bioinformatics. There has been attempts of providing semantics to workflows using OWL [7]; however, the non-tree like and bounded nature of many realistic workflows makes them more suitable for representation using description graphs.

Engineering: OWL has recently being used in engineering domains such as, for example, in the aerospace industry. These domains involve the representation of very complex structured objects, such as passenger aircrafts. The representation of such objects is specially well-suited for our formalism.

5 Technical Results

This section summarizes our main technical results on reasoning with graph-extended KBs. The first relevant result is the undecidability of the satisfiability problem: the interaction between DL axioms and rules alone, or between graphs and rules, or between DL axioms and graphs already leads to undecidability.

Theorem 1. *Checking satisfiability of graph-extended KBs* $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ *is already undecidable in the following situations:*

- $\mathcal{K} = (\mathcal{T}, \emptyset, \mathcal{G}, \mathcal{A})$ *with* $\mathcal{T}$ *a TBox in* $\mathcal{ALCF}$ *and* $\mathcal{G} = (\mathcal{G}_G, \emptyset, \emptyset)$

– $\mathcal{K} = (\emptyset, \mathcal{P}, \mathcal{G}, \mathcal{A})$ *with* $\mathcal{P}$ *a Horn program and* $\mathcal{G} = (\mathcal{G}_G, \emptyset, \emptyset)$
– $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \emptyset, \mathcal{A})$ *with* $\mathcal{P}$ *a Horn program and* $\mathcal{T}$ *in* $\mathcal{ALC}$

Undecidability is partly due to the fact that one could axiomatize models containing unbounded sequences of graphs. In practice, however, structured objects are modeled up to a certain level of granularity. Effectively, we often obtain a hierarchy of parts. For example, if the carpal bones are part of the hand, then it will most likely not be the case that the hand will eventually become a part of the carpal bones. To reflect the acyclic nature of the representation, we impose the following acyclicity condition in our formalism.

Definition 7 (Acyclic GBox). *A GBox* $\mathcal{G} = (\mathcal{G}_G, \mathcal{G}_S, \mathcal{G}_A)$ *is* acyclic *if a strict order* $\prec$ *on* $\mathcal{G}_G$ *exists s.t., for each* $G = (V, E, \lambda, M)$ *and* $G' = (V', E', \lambda', M')$ *in* $\mathcal{G}_G$*, if* $G \not\preceq G'$*, then, for each* $A \in M'$ *and* $\lhd$ *the reflexive–transitive closure of* $\lhd$ *in* $\mathcal{G}_S$*:* (i) *if* $G' \lhd G$*, then* $\neg A \in \lambda\langle i \rangle$ *for each* $i \in V \setminus V'$*;* (ii) *if* $G' \not\lhd G$*, then* $\neg A \in \lambda\langle i \rangle$ *for each* $i \in V$*.*

We call a graph-extended knowledge base *acyclic* if its GBox is acyclic. Intuitively, $G_1 \prec G_2$ means that G_2 is subordinate to G_1. In our example, we would have $G_{hand} \prec G_{finger}$ and $G_{hand} \prec G_{thumb}$, since the structures of the finger and the thumb are subordinate to the structure of a hand, respectively. We would also have $G_{finger} \prec G_{thumb}$, since a finger is more general than the thumb. The conditions in Definition 7 make it sure that no cycles occur.

Unfortunately, acyclicity is not sufficient to regain decidability To this end, we have proposed to place restrictions on the usage of atomic roles in $\mathcal{T}$, $\mathcal{P}$ and $\mathcal{G}$ in order to limit the possible interaction between different types of axioms.

Definition 8 (Role Separation). *A* role separation scheme Λ *is a triple of the form* $(N_\mathcal{T}, N_\mathcal{P}, N_\mathcal{G})$ *where* $N_\mathcal{T}$*,* $N_\mathcal{P}$*, and* $N_\mathcal{G}$ *are (not necessarily disjoint) sets of atomic roles. The roles in* $N_\mathcal{T}$*,* $N_\mathcal{P}$*, and* $N_\mathcal{G}$ *are called* $\mathcal{T}$*-,* $\mathcal{P}$*-, and* $\mathcal{G}$*-roles, respectively. A KB* $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ *is* Λ-separated *if all roles occurring in* $\mathcal{T}$*,* $\mathcal{P}$*, and* $\mathcal{G}$ *are* $\mathcal{T}$*-,* $\mathcal{P}$*-, and* $\mathcal{G}$*-roles, respectively. We say that* $\Lambda = (N_\mathcal{T}, N_\mathcal{P}, N_\mathcal{G})$ *is* weak *if* $N_\mathcal{T} \cap N_\mathcal{P} = \emptyset$*; it is* strong *if additionally* $N_\mathcal{G} = N_\mathcal{P}$*. A knowledge base* $\mathcal{K}$ *is* weakly separated *(respectively* strongly separated*) if a weak (respectively strong) role separation scheme* Λ *exists such that* $\mathcal{K}$ *is* Λ*-separated.*

Intuitively, weak separation prevents any interaction between $\mathcal{T}$ and $\mathcal{P}$. It allows one to describe general knowledge using TBox axioms and then to specialize such knowledge using graphs. For example, even if the general structure of a finger were described using DLs (e.g., this description might be a part of a general, coarse-grained KB that does not use graphs), one could describe more specialized knowledge, such as the structure of an index finger, using graphs. One can thus choose the appropriate style of modeling for knowledge at different levels of granularity. The main restriction is that one cannot use rules involving roles occuring in DL axioms. Acyclicity and weak separation seem reasonable assumptions in all the application domains mentioned in Section 4.

Strong separation restricts the modeling style in a more significant way than weak separation: essentially, it requires the modeler to determine in advance

which knowledge will be modeled using DLs and which using graphs. Thus, knowledge modeled using DLs cannot be specialized using graphs and vice versa. The restriction to strong separation is particularly limiting in the use case of chemical compounds in Section 4. It is, however, reasonable in the anatomy and engineering use cases.

Theorem 2. *Satisfiability of a weakly separated and acyclic* $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ *is* NExpTime*-complete for* $\mathcal{T}$ *in* $\mathcal{SHOQ}$ *and undecidable for* $\mathcal{T}$ *in* $\mathcal{ALCIF}$*. For* $\mathcal{K}$ *strongly separated and acyclic, satisfiability is decidable for* $\mathcal{T}$ *in* $\mathcal{SHOIQ}$*. Moreover, if* $\mathcal{T}$ *is in either* $\mathcal{SHIQ}$ *or* $\mathcal{SHOQ}$*, it is* NExpTime*-complete.*

It is worth noticing that inverse roles interact badly with graphs in the case of weak separation. In contrast, decidability is much more robust in the case of strong separation. Finally, we emphasize that it is possible to design practical reasoning algorithms for the decidable cases identified in Theorem 2. We have developed a prototypical implementation on top of the HermiT reasoner [8].[5] The preliminary evaluation we have conducted has shown promising results.

6 Future Work

The main challenge is to validate the applicability of our formalism in applications. To this end, we will extend Protégé to support description graphs and apply our formalism in the practical scenarios. We will also improve the implementation of our reasoning algorithms.

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