f-SWRL: A Fuzzy Extension of SWRL$^{(*)}$

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Abstract. In an attempt to extend existing knowledge representation systems to deal with the imperfect nature of real world information involved in several applications, the AI community has devoted considerable attention to the representation and management of uncertainty, imprecision and vague knowledge. Moreover, a lot of work has been carried out on the development of reasoning engines that can interpret imprecise knowledge. The need to deal with imperfect and imprecise information is likely to be common in the context of the (Semantic) Web. In anticipation of such requirements, this paper presents a proposal for fuzzy extensions of SWRL, which is a rule extension to OWL DL.

1 Introduction

According to widely known proposals for a Semantic Web architecture, Description Logics (DLs)-based ontologies will play a key role in the Semantic Web [Pan04]. This has led to considerable efforts to developing a suitable ontology language, culminating in the design of the OWL Web Ontology Language [BvHH$^+$04b], which is now a W3C recommendation. Although OWL adds considerable expressive power with respect to languages such as RDF, it does have expressive limitations, particularly with respect to what can be said about properties. E.g., there is no composition constructor, so it is impossible to capture relationships between a composite property and another (possibly composite) property. One way to address this problem would be to extend OWL with some form of "rules language" [HPS04]. One such proposed extension is SWRL (Semantic Web Rule Language) [HPSB$^+$04], which is a Horn clause rules extension to OWL DL[3] that overcomes many of these limitations.

Even though the combination of OWL and Horn rules results in the creation of a highly expressive language, there are still many occasions where

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[3] OWL DL is a key sub-language of OWL.

this language fails to accurately represent knowledge of our world. In particular these languages fail at representing vague and imprecise knowledge and information [Kif05]. Such type of information is apparent in many applications like multimedia processing and retrieval [SST+05,BvHH+04a], information fusion [Mat05], and many more. Experience has shown that in many cases dealing with such type of information would yield more efficient and realistic applications [AL05,ZYZ+05]. Furthermore, in many applications, like ontology alignment and modularization, the interconnection of disparate and distributed ontologies and modules is hardly ever a true or false situation, but rather a matter of a confidence or relatedness degree.

In order to capture imprecision in rules, we propose a fuzzy extension of SWRL, called f-SWRL. In f-SWRL, fuzzy individual axioms can include a specification of the "degree" (a truth value between 0 and 1) of confidence with which one can assert that an individual (resp. pair of individuals) is an instance of a given class (resp. property); and atoms in f-SWRL rules can include a "weight" (a truth value between 0 and 1) that represents the "importance" of the atom in a rule. For example, the following fuzzy rule asserts that being healthy is more important than being rich to determine if one is happy:

$$\mathsf{Rich}(?p) * 0.5 \wedge \mathsf{Healthy}(?p) * 0.9 \rightarrow \mathsf{Happy}(?p),$$

where Rich, Healthy and Happy are classes, and 0.5 and 0.9 are the weights for the atoms Rich(?p) and Healthy(?p), respectively. Additionally, observe that the classes Rich, Healthy and Happy are best represented by fuzzy concepts, since the degree to which someone is Rich is both subjective and non-crisp.

In this paper, we will present the syntax and semantics of f-SWRL. We will use standard Description Logics [BMNPS02] notations in the syntax of f-SWRL, while the model-theoretic semantics of f-SWRL is based on the theory of fuzzy sets [Zad65]. To the best of our knowledge, this is the first paper describing a fuzzy extension of the SWRL language.

2 Preliminaries

2.1 SWRL

SWRL is proposed by the Joint US/EU ad hoc Agent Markup Language Committee.[4] It extends OWL DL by introducing *rule axioms*, or simply *rules*, which have the form:

$$\mathsf{antecedent} \rightarrow \mathsf{consequent},$$

where both antecedent and consequent are conjunctions of atoms written $a_1 \wedge \ldots \wedge a_n$. Atoms in rules can be of the form C(x), P(x,y), Q(x,z), sameAs(x,y), differentFrom(x,y) or builtIn(*pred*,$\mathrm{z}_1, \ldots, \mathrm{z}_n$), where C is an OWL DL description, P is an OWL DL *individual-valued* property, Q is an OWL DL *data-valued* property, *pred* is a datatype predicate URIref, x,y are either *individual-valued*

[4] See `http://www.daml.org/committee/` for the members of the Joint Committee.

variables or OWL individuals, and z, $z_1, \ldots, z_n$ are either *data-valued* variables or an OWL data literals. An OWL data literal is either a typed literal or a plain literal; see [BvHH+04b,PH05] for details. Variables are indicated using the standard convention of prefixing them with a question mark (e.g., $?x$). For example, the following rule asserts that one's parents' brothers are one's uncles:

$$parent(?x, ?p) \wedge brother(?p, ?u) \rightarrow uncle(?x, ?u), \tag{1}$$

where $parent, brother$ and $uncle$ are all *individual-valued* properties.

In SWRL, URI references (URIrefs) are used to identify ontology elements such as classes, *individual-valued* properties and *data-valued* properties. A *URI reference* (or URIref) is a URI, together with an optional fragment identifier at the end. Uniform Resource Identifiers (URIs) are short strings that identify Web resources [Gro01]. The reader is referred to [HPSB+04] for full details of the model-theoretic semantics and abstract syntax of SWRL.

2.2 Fuzzy Sets

While in classical set theory any element belongs or not to a set, in fuzzy set theory [Zad65] this is a matter of degree. More formally, let X be a collection of elements (the universe of discourse) with cardinality m, i.e $X = \{x_1, x_2, \ldots, x_m\}$. A fuzzy subset A of X, is defined by a membership function $\mu_A(x)$, or simply $A(x)$, $x \in X$. This membership function assigns any $x \in X$ to a value between 0 and 1 that represents the degree in which this element belongs to X. The *support*, *Supp(A)*, of A is the crisp set $Supp(A) = \{x \in X \mid A(x) \neq 0\}$.

Using the above idea, the most important operations defined on crisp sets and relations (complement, union, intersection) are extended in order to cover fuzzy sets and fuzzy relations. These operations are now being performed by mathematical functions over the unit interval. More precisely, the complement $\neg A$ of a fuzzy set A is given by $(\neg A)(x) = c(A(x))$ for any $x \in X$. The intersection of two fuzzy sets A and B is given by $(A \cap B)(x) = t[A(x), B(x)]$, where t is a triangular norm (t-norm). The union of two fuzzy sets A and B is given by $(A \cup B)(x) = u[A(x), B(x)]$, where u is a triangular conorm (u-norm). A binary fuzzy relation R over two countable crisp sets X and Y is a function $R : X \times Y \rightarrow [0, 1]$. The composition of two fuzzy relation $R_1 : X \times Y \rightarrow [0, 1]$ and $R_2 : Y \times Z \rightarrow [0, 1]$ is given by $[R_1 \circ^t R_2]= \sup_{y \in Y} t[R_1(x, y), R_2(y, z)]$. Such a type of composition is referred to as sup-t composition.

Another important operation in fuzzy logics is the *fuzzy implication*, which gives a truth value to the predicate $A \Rightarrow B$. A fuzzy implication is a function ω of the form $\omega : [0, 1] \times [0, 1] \rightarrow [0, 1]$. In fuzzy logics, we are usually interested in two kinds of fuzzy implications, i.e.,

- S-implication: $\omega_{u,c}(a, b) = u(c(a), b)$,
- R-implication: $\omega_t(a, b) = \sup\{x \in [0, 1] \mid t(a, x) \leq b\}$,

where a, b are the truth values for A and B, respectively. In this paper, we use the R-implication when we define semantics of fuzzy rules (see Section 4.2).

This is because under the R-implication, and by interpreting the intersection ($\cap$) to the same t-norm (t) as that used in the ω_t operation, the truth value of $A \cap (A \Rightarrow B) \Rightarrow B$ is always 1, regardless of the truth values of a and b [Haj98,KY95], while it is not the case under the S-implication. The latter observation about S-implications can easily be verified by selecting the min(a,b) function for fuzzy intersection, the max(1-a,b) function for fuzzy implication and two arbitrary values, say 0.4 and 0.5 for a and b. The reader is referred to [KY95] for details of fuzzy logics and their applications.

3 A Motivating Use Case

In this section, we discuss a motivating use case from a casting company, which has a knowledge base that consists of person-models. Advertisement companies are using this knowledge base to look for models to be used in advertisements or other activities. Each entry in the knowledge base contains a photo of the model, personal information and some body and face characteristics. The casting company has created a user interface for inserting the information of the models as instances of a predefined ontology. It also provides a query engine to search for models with specific characteristics. A user can query the knowledge base providing high-level information about the models (such as the name, the height, the type of the hair etc.).

Now we suppose that we have only information about the following two models in the knowledge base:

- Mary has height 172cm and has weight 50kg.
- Susan has height 180cm and has weight 61kg.

If an advertisement company requires a *thin* female model. Since thinness can be regarded as a function of both the weight as well as the height of a person, one can define thinness as follows.

- One is *thin* iff one is both tall and light.
- One is *tall* iff one's height is larger than 175cm.
- One is *light* iff one's weight is less than 60kg.

Under such definitions, it is obvious that there are no thin female models in the knowledge base. Susan is over 175cm tall but is not under 60kg, while Mary is under 60kg but not over 175cm. Although Mary fails to satisfy the height requirement for only 3cm, which in fact is a rather small value, she satisfies the weight condition; in fact, she is 10kg lighter than the required weight. In fact, the advertisement company might classify her as a thin model if it regards weight a more important factor than height in terms of thinness.

The above problems can be solved if we use a fuzzy knowledge representation, instead of a crisp knowledge representation. In particular, we can define tall and light in a fuzzy way, i.e., by using degrees of confidence. For instance, based on the above data of the two models as well as the policy of the advertisement company, we can have the following fuzzy assertions.

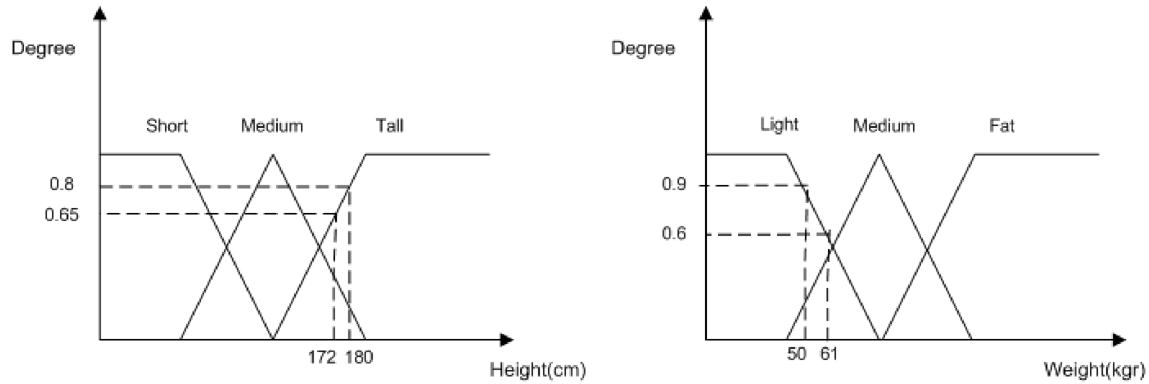

Fig. 1. The fuzzy partition of Height and Weight

- Mary is tall with a degree no less than 0.65.
- Mary is light with a degree no less than 0.9.
- Susan is tall with a degree no less than 0.8.
- Susan is light with a degree no less than 0.6.

Note that the above membership degrees of the individuals Mary and Susan to the fuzzy concepts "tall" and "light" have resulted by providing a *fuzzy partition* [KY95] of the space of the possible values that ones height and weight can obtain. For example, the fuzzy partitions in our example can be depicted in Fig. 1.

In addition to the fuzzy assertion, we can also deduce "one is thin" in a fuzzy way. For instance, we can introduce the following fuzzy rule about thinness: One is thin if one is tall (with importance factor 0.7) and light (with importance factor 0.8).

After introducing the syntax and semantics of f-SWRL, we will revisit this use case in Section 4.

4 f-SWRL

Fuzzy rules are of the form $\mathsf{antecedent} \rightarrow \mathsf{consequent}$, where atoms in both the antecedent and consequent can have weights (i.e., importance factors), i.e., numbers between 0 and 1. More specifically, atoms can be of the forms C(x)$*w$, P(x,y)$*w$, Q(x,z)$*w$, sameAs(x,y)$*w$, differentFrom(x,y)$*w$ or builtIn(pred,$z_1, \ldots, z_n$), where $w \in [0,1]$ is the weight of an atom, and omitting a weight is equivalent to specifying a value of 1. For instance, the following fuzzy rule axiom asserts that if a man has his eyebrows raised enough and his mouth open then he is happy, and that the condition that he has his eyebrows raised is a bit more important than the condition that he has his mouth open.

$$\mathsf{EyebrowsRaised}(?a) * 0.9 \wedge \mathsf{MouthOpen}(?a) * 0.8 \rightarrow \mathsf{Happy}(?a), \quad (2)$$

In this example, $\mathsf{EyebrowsRaised}, \mathsf{MouthOpen}$ and Happy are class URIrefs, ?a is a *individual-valued* variable, and 0.9 and 0.8 are the weights of the atoms $\mathsf{Eyebrows\text{-}\ Raised}$(?a) and $\mathsf{MouthOpen}$(?a), respectively.

In this paper, we only consider *atomic* fuzzy rules, i.e., rules with only one atom in the consequent. The weight of an atom in a consequent, therefore, can be seen as indicating the weight that is given to the rule axiom in determining the degree with which the consequent holds. Consider, for example, the following two fuzzy rules:

$$\mathsf{parent}(?x, ?p) \wedge \mathsf{Happy}(?p) \rightarrow \mathsf{Happy}(?x) * 0.8 \tag{3}$$

$$\mathsf{brother}(?x, ?b) \wedge \mathsf{Happy}(?b) \rightarrow \mathsf{Happy}(?x) * 0.4, \tag{4}$$

which share $\mathsf{Happy}(?x)$ in the consequent. Since $0.8 > 0.4$, more weight is given to rule (3) than to rule (4) when determining the degree to which an individual is Happy.

In what follows, we formally introduce the syntax and model-theoretic semantics of fuzzy SWRL.

4.1 Syntax

In this section, we present the syntax of fuzzy SWRL. To make the presentation simple and clear, we use DL syntax (see the following definition) instead of the XML, RDF or abstract syntax of SWRL.

Definition 1. *Let* $\mathtt{a,b}$ *be individual URIrefs,* l *a OWL data literal,* C, D *OWL class descriptions,* r, s *OWL* individual-valued *property descriptions,* r_1, r_2 individual-valued *property URIrefs,* s, s_1 data-valued *property URIrefs, pred a datatype predicate,* $m_1, m_2, m_3, w, w_1, \ldots, w_n \in [0, 1]$, $\vec{v}, \vec{v}_1, \ldots, \vec{v}_n$ *are (unary or binary) tuples of variables and/or individual URIrefs,* $a_1(\vec{v_1}), \ldots, a_n(\vec{v_n})$ *and* $c(\vec{v})$ *are of the forms* $C(x)$, $r(x, y)$, $s(x, z)$, $sameAs(x, y)$, $differentFrom(x, y)$ *or builtIn(pred,*$z_1, \ldots, z_n$*), where* x, y *are* individual-valued *variables or individual URIrefs and* z, $z_1, \ldots, z_n$ *are* data-valued *variables or OWL data literals.*

An f-SWRL ontology can have the following kinds of axioms:

- *class axioms:* $C \sqsubseteq D$ *(class inclusion axioms);*
- *property axioms:* $r \sqsubseteq r_1$ *(*individual-valued *property inclusion axioms),* $\mathsf{Func}(r_1)$ *(functional* individual-valued *property axioms),* $\mathsf{Trans}(r_2)$ *(transitive property axioms),* $s \sqsubseteq s_1$ *(*data-valued *property inclusion axioms),* $\mathsf{Func}(s_1)$ *(functional* data-valued *property axioms);*
- *individual axioms:* $(\mathtt{a} : C) \geq m_1$ *(fuzzy class assertions),* $(\langle \mathtt{a}, \mathtt{b} \rangle : r) \geq m_2$ *(fuzzy* individual-valued *property assertions),* $(\langle \mathtt{a}, \mathtt{l} \rangle : r) \geq m_3$ *(fuzzy* data-valued *property assertions),* $\mathtt{a} = \mathtt{b}$ *(individual equality axioms) and* $\mathtt{a} \neq \mathtt{b}$ *(individual inequality axioms);*
- *rule axioms:* $a_1(\vec{v_1}) * w_1 \wedge \cdots \wedge a_n(\vec{v_n}) * w_n \rightarrow c(\vec{v}) * w$ *(fuzzy rule axioms).*

Omitting a degree or a weight is equivalent to specifying the value of 1. ◇

According to the above definition, f-SWRL extends SWRL with fuzzy class assertions, fuzzy property assertions and fuzzy rule axioms.

4.2 Model-theoretic Semantics

In this section, we give a model-theoretic semantics for fuzzy SWRL. Although many f-SWRL axioms share the same syntax as their counterparts in SWRL, such as concept inclusion axioms, they have different semantics because we use fuzzy interpretations in the model-theoretic semantics of f-SWRL.

Before we provide a model-theoretic semantics for f-SWRL, we introduce the notions of datatype predicates and datatype predicate maps.

Definition 2. (Datatype Predicate) *A* datatype predicate *(or simply* predicate*) p is characterised by an arity $a(p)$, or a minimum arity $a_{min}(p)$ if p can have multiple arities, and a predicate extension (or simply* extension*) $E(p)$.* ◇

For example, $=^{int}$ is a datatype predicate with arity $a(=^{int}) = 2$ and extension $E(=^{int}) = \{\langle i_1, i_2\rangle \in V(integer)^2 \mid i_1 = i_2\}$, where $V(integer)$ is the set of all integers. Datatypes can be regarded as *special* predicates with arity 1 and predicate extensions equal to their value spaces; e.g., the datatype *integer* can be seen as a predicate with arity $a(integer) = 1$ and predicate extension $E(integer) = V(integer)$.[5]

Definition 3. (Predicate Map) *We consider a predicate map $\mathbf{M}_p$ that is a partial mapping from predicate URI references to predicates.* ◇

Intuitively, datatype predicates (resp. datatype predicate URIrefs) in $\mathbf{M}_p$ are called built-in datatype predicates (resp. datatype predicate URIrefs) w.r.t. $\mathbf{M}_p$. Note that allowing the datatype predicate map to vary allows different implementations of f-SWRL to implement different datatype predicates.

Definition 4. *Given a datatype predicate map $\mathbf{M}_p$, a fuzzy interpretation is a triple $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \Delta_{\mathbf{D}}, \cdot^{\mathcal{I}}\rangle$, where the abstract domain $\Delta^{\mathcal{I}}$ is a non-empty set, the datatype domain contains at least all the data values in the extensions of built-in datatype predicates in $\mathbf{M}_p$, and $\cdot^{\mathcal{I}}$ is a fuzzy interpretation function, which maps*

1. *individual URIref and* individual-valued *variables to elements of $\Delta^{\mathcal{I}}$,*
2. *a class description C to a membership function $C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0, 1]$,*
3. *an* individual-valued *property URIref r to a membership function $r^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$,*
4. *an* data-valued *property URIref q to a membership function $q^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta_{\mathbf{D}} \to [0, 1]$,*
5. *a built-in datatype predicate URIref pred to its extension $pred^{\mathcal{I}} = E(\mathbf{M}_p(pred)) \in (\Delta_{\mathbf{D}})^n$, where $n = a(\mathbf{M}_p(pred))$, so that*

$$builtIn^{\mathcal{I}}(pred, z_1, \ldots, z_n) = \begin{cases} 1 \text{ if } \langle z_1^{\mathcal{I}}, \ldots, z_n^{\mathcal{I}}\rangle \in pred^{\mathcal{I}} \\ 0 \text{ otherwise,} \end{cases}$$

[5] See [Pan04] for detailed discussion on the relationship between datatypes and datatype predicates.

DL Syntax	Semantics
A	$A^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1]$
$\top$	$\top^{\mathcal{I}}(a) = 1$
$\bot$	$\bot^{\mathcal{I}}(a) = 0$
$C_1 \sqcap C_2$	$(C \sqcap D)^{\mathcal{I}}(a) = t(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a))$
$C_1 \sqcup C_2$	$(C \sqcup D)^{\mathcal{I}}(a) = u(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a))$
$\neg C$	$(\neg C)^{\mathcal{I}}(a) = c(C^{\mathcal{I}}(a))$
$\{o_1\}\sqcup \{o_2\}$	$(\{o_1\} \sqcup \{o_2\})^{I}(a) = 1$ if $a \in \{ o_1^I, o_2^I\}$
	$(\{o_1\} \sqcup \{o_2\})^{I}(a) = 0$ otherwise
$\exists r.C$	$(\exists r.C)^{\mathcal{I}}(a) = \sup_{b\in\Delta^{\mathcal{I}}} t(r^{\mathcal{I}}(a,b), C^{\mathcal{I}}(b))$
$\forall r.C$	$(\forall r.C)^{\mathcal{I}}(a) = \inf_{b\in\Delta^{\mathcal{I}}} \omega_t(r^{\mathcal{I}}(a,b), C^{\mathcal{I}}(b))$
$\exists r.\{o\}$	$(\exists(r.\{o\})^{\mathcal{I}}(a) = \sup_{b\in\Delta^{\mathcal{I}}} t(r^{\mathcal{I}}(a,b), \{o\}^{\mathcal{I}}(b))$
$\geqslant mr$	$(\geqslant mr)^{\mathcal{I}}(a) = 1$ if $\mid Supp[r^{\mathcal{I}}(a,b)] \mid \geq m\}$
	$(\geqslant mr)^{\mathcal{I}}(a) = 0$ otherwise
$\leqslant mr$	$(\leqslant mr)^{\mathcal{I}}(a) = 1$ if $\mid Supp[r^{\mathcal{I}}(a,b)] \mid \leq m\}$
	$(\geqslant mr)^{\mathcal{I}}(a) = 0$ otherwise
$\exists s.d$	$(\exists s.d)^{\mathcal{I}}(a) = \sup_{y\in\Delta_{\mathbf{D}}} t(s^{\mathcal{I}}(a,y), y \in d^{\mathcal{I}})$
$\forall s.d$	$(\forall s.d)^{\mathcal{I}}(a) = \inf_{y\in\Delta_{\mathbf{D}}} \omega_t(s^{\mathcal{I}}(a,y), y \in d^{\mathcal{I}})$
$\geqslant ms$	$(\geqslant ms)^{\mathcal{I}}(a) = 1$ if $\mid Supp[s^{\mathcal{I}}(a,y)] \mid \geq m\}$
	$(\geqslant ms)^{\mathcal{I}}(a) = 0$ otherwise
$\leqslant ms$	$(\leqslant ms)^{\mathcal{I}}(a) = 1$ if $\mid Supp[s^{\mathcal{I}}(a,y)] \mid \leq m\}$
	$(\geqslant ms)^{\mathcal{I}}(a) = 0$ otherwise

Table 1. Syntax and Semantics of Fuzzy Class Descriptions

6. *the built-in property sameAs to a membership function*

$$sameAs^{\mathcal{I}}(x,y) = \begin{cases} 1 \text{ if } x^{\mathcal{I}} = y^{\mathcal{I}} \\ 0 \text{ otherwise,} \end{cases}$$

7. *the built-in property $differentFrom$ to a membership function*

$$differentFrom^{\mathcal{I}}(x,y) = \begin{cases} 1 \text{ if } x^{\mathcal{I}} \neq y^{\mathcal{I}} \\ 0 \text{ otherwise.} \end{cases}$$

The fuzzy interpretation function can be extended to give semantics for fuzzy concept descriptions listed in Table 1 (where $\mid \cdot \mid$ denotes cardinality).

A fuzzy interpretation $\mathcal{I}$ satisfies a class inclusion axiom $C \sqsubseteq D$, written $\mathcal{I} \models C \sqsubseteq D$, if $\forall o \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(o) \leq D^{\mathcal{I}}(o)$.

A fuzzy interpretation $\mathcal{I}$ satisfies an individual-valued *property inclusion axiom $r \sqsubseteq r_1$, written $\mathcal{I} \models r \sqsubseteq r_1$, if $\forall o,q \in \Delta^{\mathcal{I}}, r^{\mathcal{I}}(o,q) \leq r_1^{\mathcal{I}}(o,q)$. $\mathcal{I}$ satisfies a functional* individual-valued *property axiom* $\mathsf{Func}(r_1)$, *written $\mathcal{I} \models \mathsf{Func}(r_1)$, if $\forall o,q \in \Delta^{\mathcal{I}}, \mid Supp[r_1^{\mathcal{I}}(o,q)] \mid \leq 1$. $\mathcal{I}$ satisfies a transitive property axiom* $\mathsf{Trans}(r_2)$, *written $\mathcal{I} \models \mathsf{Trans}(r_2)$, if $\forall o,q \in \Delta^{\mathcal{I}}, r_2^{\mathcal{I}}(o,q) = \sup_{p\in\Delta^{\mathcal{I}}} t[r_2^{\mathcal{I}}(o,p), r_2^{\mathcal{I}}(p,q)]$, where t is a triangular norm. A fuzzy interpretation $\mathcal{I}$ satisfies a* data-valued *property inclusion axiom $s \sqsubseteq s_1$, written $\mathcal{I} \models s \sqsubseteq s_1$, if $\forall \langle o,l \rangle \in \Delta^{\mathcal{I}} \times$*

$\Delta_{\mathbf{D}}, s^{\mathcal{I}}(o,l) \leq s_1^{\mathcal{I}}(o,l)$. *$\mathcal{I}$ satisfies a functional* data-valued *property axiom* $\mathsf{Func}(s_1)$, *written* $\mathcal{I} \models \mathsf{Func}(s_1)$, *if* $\forall \langle o,l \rangle \in \Delta^{\mathcal{I}} \times \Delta_{\mathbf{D}}, \mid Supp[s_1^{\mathcal{I}}(o,l)] \mid \leq 1$.

A fuzzy interpretation $\mathcal{I}$ satisfies a fuzzy class assertion $(\mathtt{a} : C) \geq m$, *written* $\mathcal{I} \models (\mathtt{a} : C) \geq m$, *if* $C^{\mathcal{I}}(\mathtt{a}) \geq m$. *$\mathcal{I}$ satisfies a fuzzy* individual-valued *property assertion* $(\langle \mathtt{a},\mathtt{b} \rangle : r) \geq m_2$, *written* $\mathcal{I} \models (\langle \mathtt{a},\mathtt{b} \rangle : r) \geq m_2$, *if* $r^{\mathcal{I}}(\mathtt{a},\mathtt{b}) \geq m_2$. *$\mathcal{I}$ satisfies a fuzzy* data-valued *property assertion* $(\langle \mathtt{a}, l \rangle : s) \geq m_3$, *written* $\mathcal{I} \models (\langle \mathtt{a}, l \rangle : s) \geq m_3$, *if* $s^{\mathcal{I}}(\mathtt{a}, l) \geq m_3$. *$\mathcal{I}$ satisfies an individual equality axiom* $\mathtt{a} = \mathtt{b}$, *written* $\mathcal{I} \models \mathtt{a} = \mathtt{b}$, *if* $\mathtt{a}^{\mathcal{I}} = \mathtt{b}^{\mathcal{I}}$. *$\mathcal{I}$ satisfies an individual inequality axiom* $\mathtt{a} \neq \mathtt{b}$, *written* $\mathcal{I} \models \mathtt{a} \neq \mathtt{b}$, *if* $\mathtt{a}^{\mathcal{I}} \neq \mathtt{b}^{\mathcal{I}}$.

A fuzzy interpretation $\mathcal{I}$ satisfies a fuzzy rule axiom $a_1(\overrightarrow{v_1}) * w_1 \wedge \cdots \wedge a_n(\overrightarrow{v_n}) * w_n \rightarrow c(\overrightarrow{v}) * w$, *written* $\mathcal{I} \models a_1(\overrightarrow{v_1}) * w_1 \wedge \cdots \wedge a_n(\overrightarrow{v_n}) * w_n \rightarrow c(\overrightarrow{v}) * w$, *if* $t(t(a_1^{\mathcal{I}}(\overrightarrow{v_1}^{\mathcal{I}}), w_1), \ldots, t(a_n^{\mathcal{I}}(\overrightarrow{v_n}^{\mathcal{I}}), w_n)) \leq t(c^{\mathcal{I}}(\overrightarrow{v}^{\mathcal{I}}), w)$, *where t is a triangular norm.* $\diamond$

There are some remarks on the above definition. Firstly, we use R-implication for fuzzy rule axioms. Recall from Section 2.2 that in R-implication, $\omega_t(a,b) = \sup\{x \in [0,1] \mid t(a,x) \leq b\}$. A fuzzy interpretation $\mathcal{I}$ satisfies a rule axiom $\mathsf{antecedent} \rightarrow \mathsf{consequent}$, if $\omega_t(\mathsf{d}_{\mathsf{antecedent}}, \mathsf{d}_{\mathsf{consequent}}) = 1$. It is easy to show that $\omega_t(\mathsf{d}_{\mathsf{antecedent}}, \mathsf{d}_{\mathsf{consequent}}) = 1$ if and only if $\mathsf{d}_{\mathsf{antecedent}} \leq \mathsf{d}_{\mathsf{consequent}}$. Indeed, if $\mathsf{d}_{\mathsf{antecedent}} \leq \mathsf{d}_{\mathsf{consequent}}$, we have $t(\mathsf{d}_{\mathsf{antecedent}}, 1) \leq \mathsf{d}_{\mathsf{consequent}}$. Hence, we have $\omega_t(\mathsf{d}_{\mathsf{antecedent}}, \mathsf{d}_{\mathsf{consequent}}) = 1$. On the other hand, if $\omega_t(\mathsf{d}_{\mathsf{antecedent}}, \mathsf{d}_{\mathsf{consequent}}) = 1$, then $t(\mathsf{d}_{\mathsf{antecedent}}, 1) \leq \mathsf{d}_{\mathsf{consequent}}$. Due to the boundary condition of t-norms (i.e., $t(a,1) = a$), we have $\mathsf{d}_{\mathsf{antecedent}} \leq \mathsf{d}_{\mathsf{consequent}}$. Now let us take the rule (2) as an example to illustrate the above semantics on fuzzy rule axioms. Assuming that EyebrowsRaised, MouthOpen and Happy are class URIrefs, then given a fuzzy interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, the rule (2) is satisfied by $\mathcal{I}$ iff for all $a \in \Delta^{\mathcal{I}}$, we have

$$t(t(\mathsf{EyebrowsRaised}^{\mathcal{I}}(a), 0.9), t(\mathsf{MouthOpen}^{\mathcal{I}}(a), 0.8)) \leq t(\mathsf{Happy}^{\mathcal{I}}(a), 1).$$

Secondly, there is more than one choice of semantics of fuzzy class descriptions. The one we presented in Table 1 is simply a relatively straight forward one out of many possible choices. For example, we decide to use R-implication in value restriction ($\forall r.C$) and datatype value restriction ($\forall s.d$) because we use R-implication in fuzzy rule axioms.

Thirdly, we have the following equivalence between the f-SWRL individual axiom and rule axiom: The fuzzy assertion $(\mathtt{Tom} : \mathsf{Happy}) \geq 0.8$ is equivalent to the rule axiom $\top(\mathtt{Tom}) * 0.8 \rightarrow \mathsf{Happy}(\mathtt{Tom})$. According to the above semantics, we have:

$$t(\top^{\mathcal{I}}(\mathtt{Tom}), 0.8) \leq \mathsf{Happy}^{\mathcal{I}}(\mathtt{Tom})$$

From a semantics point of view an individual always belong to a degree of 1 to the top concept, so we have:

$$t(1, 0.8) \leq \mathsf{Happy}^{\mathcal{I}}(\mathtt{Tom})$$

Due to the boundary condition of t-norms, we have

$$\mathsf{Happy}^{\mathcal{I}}(\texttt{Tom}) \geq 0.8.$$

This suggests that fuzzy assertion can be represented by fuzzy rule axioms.

Last but not least, suppose we have the following *chaining* of rules: $\mathsf{B}_1 \rightarrow \mathsf{H}_1$ and $\mathsf{H}_1 \rightarrow \mathsf{H}_2$. If a fuzzy interpretation $\mathcal{I}$ satisfy these fuzzy rule axioms, $t_{\mathsf{B}_1}(\mathcal{I}) \leq t_{\mathsf{H}_1}(\mathcal{I})$ and $t_{\mathsf{B}_2}(\mathcal{I}) \leq t_{\mathsf{H}_2}(\mathcal{I})$. It follows that $t_{\mathsf{B}_1}(\mathcal{I}) \leq t_{\mathsf{H}_2}(\mathcal{I})$; thus, $\mathcal{I}$ also satisfies the rule $\mathsf{B}_1 \rightarrow \mathsf{H}_2$. Note that we abuse the notation here and use, for example, $t_{\mathsf{B}_1}(\mathcal{I})$ to represent the t-norm of B_1 given the fuzzy interpretation $\mathcal{I}$.

Example 1. Now we revised the use case we presented in Section 3. The f-SWRL knowledge base about models consists of the following fuzzy axioms:

- Mary is Tall with a degree no less than 0.65: $(\texttt{Mary} : \mathsf{Tall}) \geq 0.65$.
- Mary is Light with a degree no less than 0.9: $(\texttt{Mary} : \mathsf{Light}) \geq 0.9$.
- Susan is Tall with a degree no less than 0.8: $(\texttt{Susan} : \mathsf{Tall}) \geq 0.8$.
- Susan is Light with a degree no less than 0.6: $(\texttt{Susan} : \mathsf{Light}) \geq 0.6$.
- One is Thin if one is Tall (with importance factor 0.7) and Light (with importance factor 0.8):

$$\mathsf{Tall}(?p) * 0.7 \wedge \mathsf{Light}(?p) * 0.8 \rightarrow \mathsf{Thin}(?p).$$

According to Definition 4, if we use the *min* t-norm, we have $\mathsf{Thin}^{\mathcal{I}}(\texttt{Mary}^{\mathcal{I}}) \geq 0.65$ and $\mathsf{Thin}^{\mathcal{I}}(\texttt{Susan}^{\mathcal{I}}) \geq 0.6$. ♢

5 Discussion

Several ways of extending Description Logics using the theory of fuzzy logic have been proposed in the literature [Yen91,TM98,Str01,Str05,SST+05]. Furthermore, in [Str04] an approach to extend *Description Logic Programs* (DLPs) with uncertainty was provided, where DLP is extended with *negation as failure*, which is not supported by SWRL. In [Voj01] an approach to fuzzy logic programs similar to ours was provided. It used Herbrand models, instead of model theoretic ones, and it did not include weights to rule atoms. To the best of our knowledge, there exists no publication on fuzzy extensions of SWRL. We believe that the combination of Semantics Web ontology and rules languages provides a powerful and flexible knowledge representation formalism, and that f-SWRL is of great interest to the ontology community as well as to communities in which ontologies with vague information can be applied, such as multimedia and the Semantic Web.

Our future work includes logical properties and computational aspect of f-SWRL. Another interesting direction is to extend f-SWRL to support datatype groups [Pan04], which allows the use of customised datatypes and datatype predicates in ontologies.

References

[AL05] S. Agarwal and S. Lamparter. smart-a semantic matchmaking portal for electronic markets. In *International World Wide Web Conference Committee*, 2005.

[BMNPS02] F. Baader, D. L. McGuiness, D. Nardi, and P. Patel-Schneider, editors. *Description Logic Handbook: Theory, implementation and applications*. Cambridge University Press, 2002.

[BvHH+04a] Sean Bechhofer, Frank van Harmelen, James Hendler, Ian Horrocks, Deborah L. McGuinness, Peter F. Patel-Schneider, and Lynn Andrea Stein eds. OWL Web Ontology Language Reference, Feb 2004.

[BvHH+04b] Sean Bechhofer, Frank van Harmelen, Jim Hendler, Ian Horrocks, Deborah L. McGuinness, Peter F. Patel-Schneider, Lynn Andrea Stein; Mike Dean, and Guus Schreiber (editors). OWL Web Ontology Language Reference. Technical report, W3C, February 10 2004. `http://www.w3.org/TR/2004/REC-owl-ref-20040210/`.

[Gro01] Joint W3C/IETF URI Planning Interest Group. URIs, URLs, and URNs: Clarifications and Recommendations 1.0. URL `http://www.w3.org/TR/uri-clarification/`, 2001. W3C Note.

[Haj98] P. Hajek. *Metamathematics of fuzzy logic*. Kluwer, 1998.

[HPS04] Ian Horrocks and Peter F. Patel-Schneider. A Proposal for an OWL Rules Language. In *Proc. of the Thirteenth International World Wide Web Conference (WWW 2004)*, pages 723–731. ACM, 2004.

[HPSB+04] Ian Horrocks, Peter F. Patel-Schneider, Harold Boley, Said Tabet, Benjamin Grosof, and Mike Dean. SWRL: A Semantic Web Rule Language — Combining OWL and RuleML. W3C Member Submission, `http://www.w3.org/Submission/SWRL/`, May 2004.

[Kif05] Michael Kifer. Requirements for an expressive rule language on the semantic web. W3C Workshop on Rule Languages for Interoperability, 2005.

[KY95] G. J. Klir and B. Yuan. *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Prentice-Hall, 1995.

[Mat05] Christopher J. Matheus. Using ontology-based rules for situation awareness and information fusion. W3C Workshop on Rule Languages for Interoperability, 2005.

[Pan04] Jeff Z. Pan. *Description Logics: Reasoning Support for the Semantic Web*. PhD thesis, School of Computer Science, The University of Manchester, Oxford Rd, Manchester M13 9PL, UK, Sept 2004.

[PH05] Jeff Z. Pan and Ian Horrocks. OWL-Eu: Adding Customised Datatypes into OWL. In *Proc. of Second European Semantic Web Conference (ESWC 2005)*, 2005. To appear.

[SST+05] G. Stoilos, G. Stamou, V. Tzouvaras, J.Z. Pan, and I. Horrocks. A fuzzy description logic for multimedia knowledge representation. Proc. of the International Workshop on Multimedia and the Semantic Web, 2005.

[Str01] U. Straccia. Reasoning within fuzzy description logics. *Journal of Artificial Intelligence*, 14:137–166, 2001.

[Str04] U. Straccia. Uncertainty and description logic programs: A proposal for expressing rules and uncertainty on top of ontologies. Technical report, Pisa, Italy, 2004.

[Str05] Umberto Straccia. Towards a fuzzy description logic for the semantic web (preliminary report). In *2nd European Semantic Web Conference (ESWC-05)*, Lecture Notes in Computer Science, Crete, 2005. Springer Verlag.

[TM98] C. Tresp and R. Molitor. A description logic for vague knowledge. In *In proc of the 13th European Conf. on Artificial Intelligence (ECAI-98)*, 1998.

[Voj01] P. Vojtás. Fuzzy logic programming. *Fuzzy Sets and Systems*, 124:361–370, 2001.

[Yen91] J. Yen. Generalising term subsumption languages to fuzzy logic. In *In Proc of the 12th Int. Joint Conf on Artificial Intelligence (IJCAI-91)*, pages 472–477, 1991.

[Zad65] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

[ZYZ+05] L. Zhang, Y. Yu, J. Zhou, C. Lin, and Y. Yang. An enhanced model for searching in semantic portals. In *International World Wide Web Conference Committee*, 2005.