Performance of DLP on Random Modal Formulae

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A system that correctly reasons with information in an expressive description logic includes a decision procedure for an expressive propositional modal logic. This decision procedure must be heavily optimised if it is to be able to usefully reason with knowledge bases of any complexity. Description logic systems that incorporate a heavily-optimised propositional modal logic decision procedure include FACT [5], DLP [7], and HAM-ALC [4].

Therefore, a system that efficiently reasons with information in an expressive description logic can be used as a fast decision procedure for an expressive propositional modal logic. The current status is that not only are the fastest reasoners for expressive description logics fast reasoners for propositional modal logics, but for many classes of formulae they are the *fastest* such reasoners.

We have performed numerous experiments with DLP, showing that it is competitive with other reasoners for propositional modal logics, including comparisons presented at recent Tableaux conferences where DLP was the fastest system [1, 8]. In more recent tests we have compared DLP with KSAT [3], TA [6], and KSATC [2] on various collections of random formulae.

The two fastest of these systems by a considerable amount are DLP and KSATC. DLP is an experimental description logic system available from Bell Labs at `http://www.bell-labs.com/user/pfps/dlp`. It implements a very expressive description logic, including full regular expressions on roles. KSATC is a reasoner for $\mathbf{K_{(m)}}$, built on a fast Davis-Putnam-Logemann-Loveland decision procedure for propositional logic. KSATC is available at `ftp://ftp.mrg.dist.unige.it/pub/mrg-systems/KR98-sources/KSat-source/KSatC`.

Tests illustrating the differences between DLP and KSATC are reported in Figures 1 and 2. Both figures give results for randomly-generated 3CNF formulae in

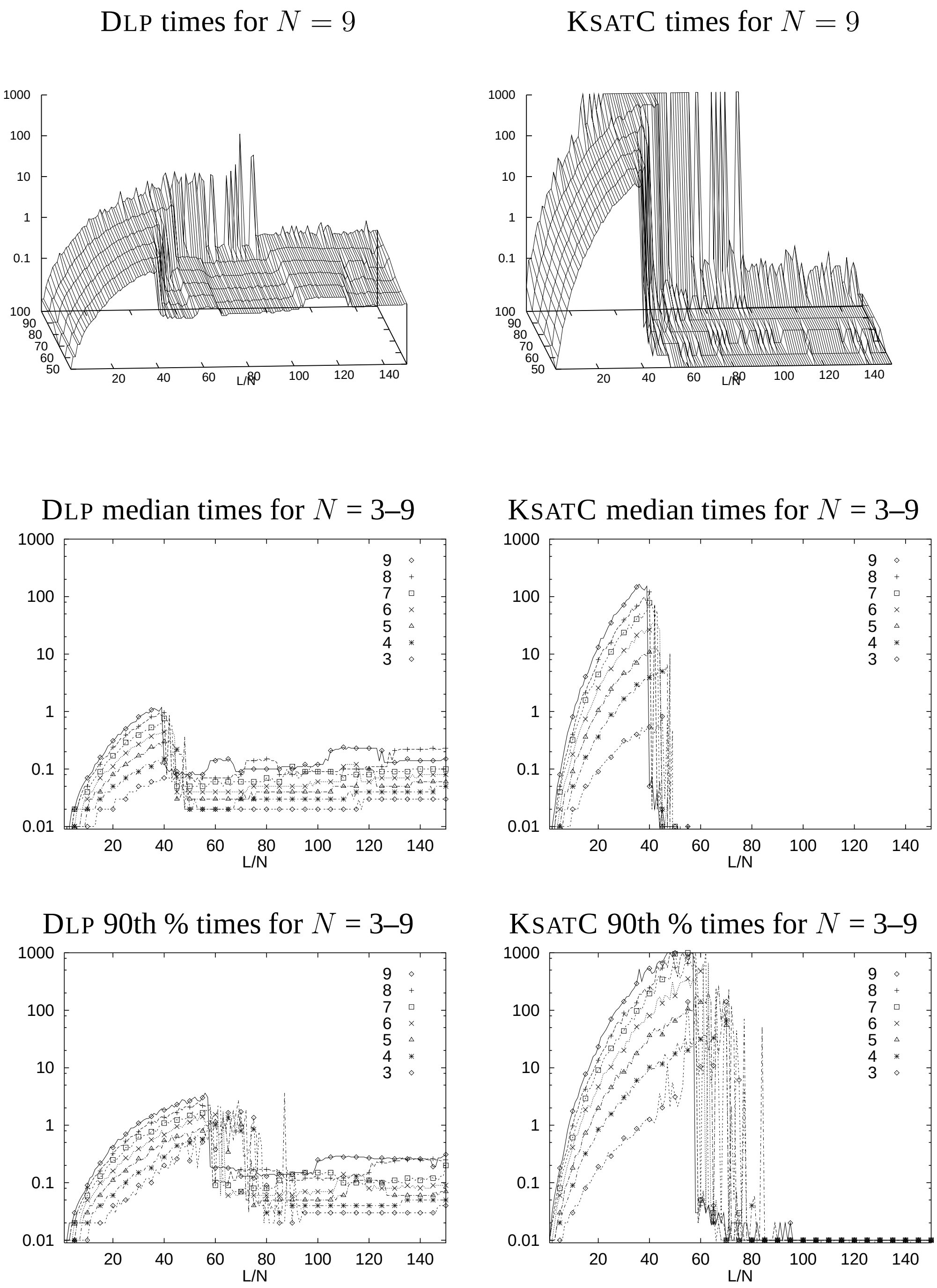

Figure 1: Results for test 1 (modal depth 2)

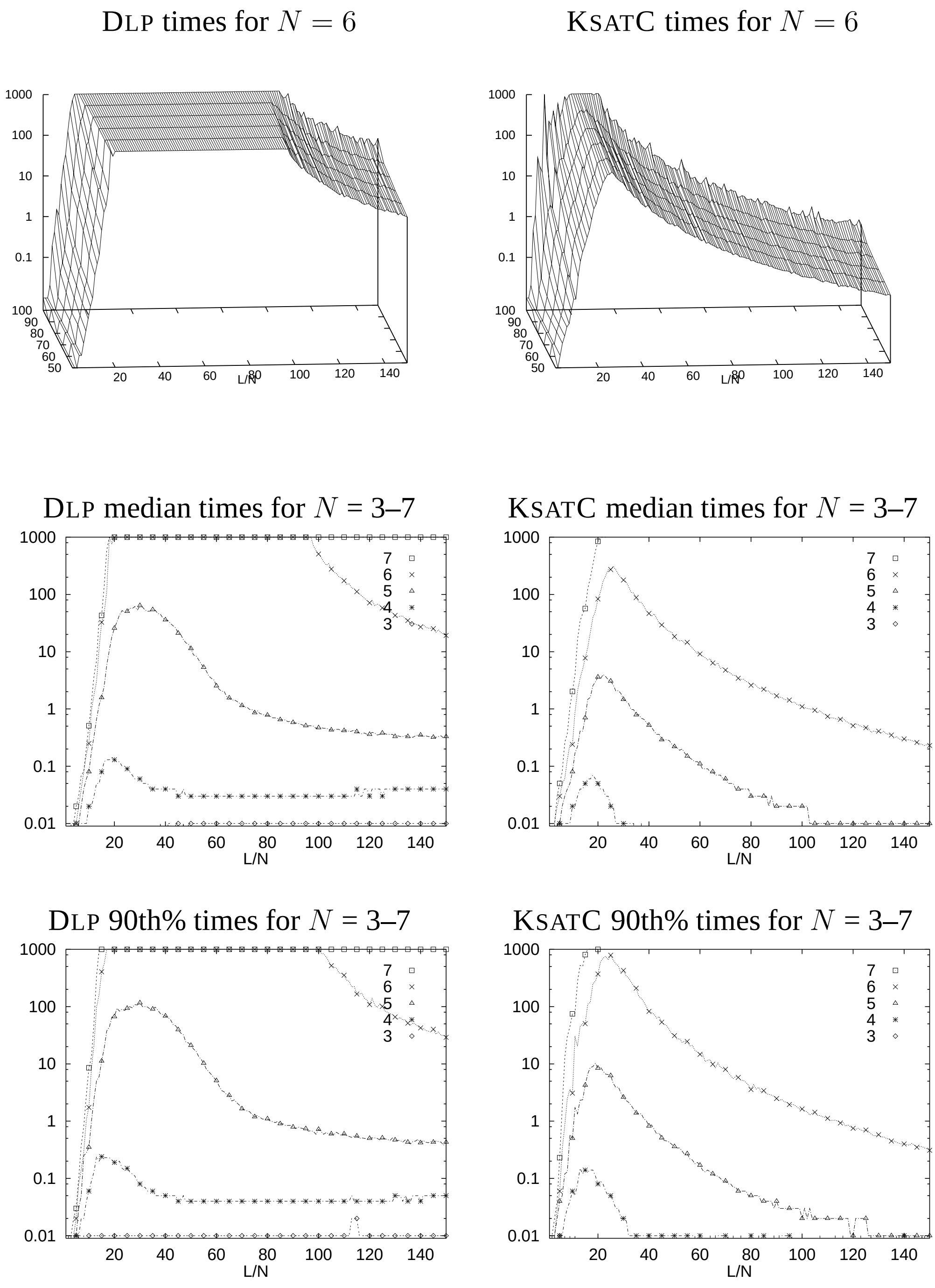

Figure 2: Results for test 2 (modal depth 1)

the modal logic **K** using a formula generator that we have devised, which is similar to previously-used formula generators but eliminates their problems.

Figure 1 gives the results for formulae with a maximum modal depth (role restriction depth) of 2. For a given number of propositional variables (N), ranging from 3 to 9, formulae were generated with a number of clauses ranging from the number of propositional variables up to 150 times this number (shown as L/N in the results). In each clause there was a 50% chance that an atom was a propositional variable or modal formulae at depths 0 and 1.

For large values of L/N in these tests KSATC often takes very little time (less than 0.01 seconds) whereas DLP takes considerably longer (about 0.1 seconds). These formulae are unsatisfiable even ignoring any modal subformulae. Both DLP and KSAT can easily determine this sort of unsatisfiability. Because the DPLL algorithm in KSAT can perform single passes over a large set of clauses in a very short time, KSAT can process the trivially unsatisfiable formulae very quickly, but DLP's data structures and low-level algorithms are not heavily optimised so DLP takes a total time for such formulae of about 0.1 seconds.

For data points in the L/N range from about 20–50, many or most of the formulae are satisfiable. Here DLP is uniformly faster than KSATC. Much of this difference is due to KSATC investigating modal successors (role restrictions) at every choice point. Modal successors do not produce much unsatisfiability in these tests, so the investigation of modal successors does not cut off search.

Even outside the above ranges (of L/N values), DLP performs better than KSATC for *satisfiable* formulae. Although there may be some benefit in examining modal successors early for these formulae, this benefit would occur only when a large number of necessary modal formulae have been assigned true. Because of the repeated work performed by KSATC on such successors, it does not appear that the benefits are realized.

Figure 2 gives the results for formulae with a maximum modal depth (role restriction depth) of 1. Here all atoms at depth 0 are modal formulae. Here KSATC performs much better than DLP. Because it aggressively investigates the modal successors, it can eliminate search at the top level. DLP has to examine many modal successors, as it will generate many top-level assignments that contain nearly the same modal successors. Its caching of modal results does not help very much as there are so many different modal successors that can be generated.

We have performed other tests using the same generator showing some areas where DLP dominates, some where KSATC dominates, and some where they perform equally well.

Even though we have performed many of these test using randomly-generated formulae, we are not happy with the fact that we are restricted to such tests. We would much prefer to test the performance of DLP on real knowledge bases that use the more-powerful features of DLP's description logic. However, such knowledge bases do not yet exist. We hope that the performance of DLP, and of other expressive description logic systems, will encourage people to develop knowledge bases using these features. We are highly motivated to work with developers of such knowledge bases.

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