Extending Consequence-Based Reasoning to $\mathcal{SRIQ}$

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Motivation

- Most reasoners based on (hyper)tableau
  - FaCT++ – HermiT – Pellet – Konclude – Racer
- Work reasonably well in practice
- But building many counter models is expensive
  - To prove $\mathcal{O} \models C \sqsubseteq D$ show $C \sqcap \neg D$ is unsat
  - Bottleneck: large number of concepts
  - Rebuilds entire model for each test

Consequence-based Features

Optimal worse-case complexity

One pass classification

No need for several counter models

Pay as you go

Deterministic

State of the art

ELK (Java)

Snorocket (Java)

$\mathcal{EL}$

CEL (Common LISP)

Elephant (C)

jcel (Java)

Horn-$\mathcal{SHIQ}$

CB (OCaml)

Horn-$\mathcal{SROIQ}$

$\mathcal{ALCI}$

$\mathcal{ALCH}$

$\mathcal{SRIQ}$

Condor (C++)

Key Facts

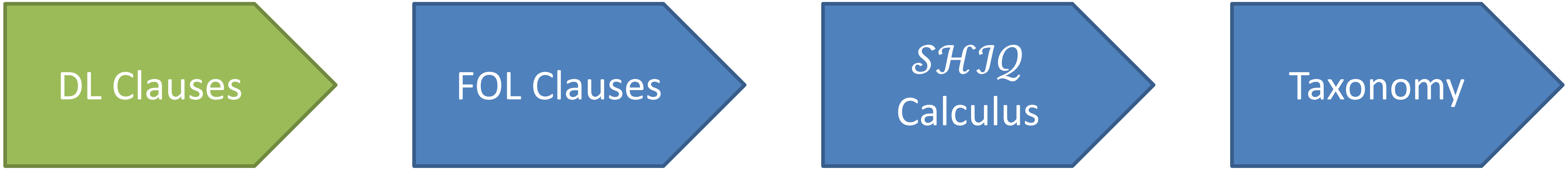
① Algorithm does not build models

→ Apply inference rules to derive local consequences of ontology

② Derived consequences not all stored together

→ Contexts store consequences corresponding to a conjunction of concepts and roles

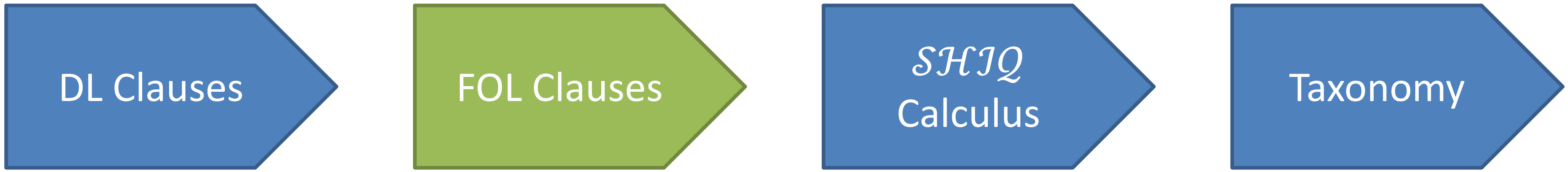
Reasoning Stages



Example

$$\begin{aligned}
\mathrm{Vegetarian} &\sqsubseteq \mathrm{Animal} \\
\mathrm{Animal} &\sqsubseteq \geq 5\, \mathrm{eats} \\
\mathrm{Meat} \sqcap \mathrm{SideDish} &\sqsubseteq \bot \\
\mathrm{Vegetarian} &\sqsubseteq \forall\, \mathrm{eats} . \mathrm{SideDish} \\
\geq 5\, \mathrm{eats} . \neg \mathrm{Meat} &\sqsubseteq \mathrm{HealthyPerson} \\
\mathrm{HealthyPerson} &\sqsubseteq \mathrm{Person}
\end{aligned}$$

$\mathrm{Vegetarian} \sqsubseteq \mathrm{Person}$?


DL Clauses
FOL Clauses
$\mathcal{SHIQ}$ Calculus
Taxonomy

Structural transformation

Translate into first-order clauses with equality

$$\bigwedge_{i=1}^{n} T_i \rightarrow \bigvee_{j=1}^{m} L_j$$

Atoms or equations of the forms
$z_i \approx z_j$ or $z_i \not\approx z_j$

Function-free atoms of the forms
$A(x), R(x, z_i)$ or $R(z_i, x)$

$$
\begin{aligned}
\mathsf{Vegetarian} &\sqsubseteq \mathsf{Animal} \\
\mathsf{Animal} &\sqsubseteq \geq 5\ \mathsf{eats} \\
\mathsf{Meat} \sqcap \mathsf{SideDish} &\sqsubseteq \bot \\
\mathsf{Vegetarian} &\sqsubseteq \forall\ \mathsf{eats}\,.\,\mathsf{SideDish} \\
\geq 5\ \mathsf{eats}\,.\,\neg\mathsf{Meat} &\sqsubseteq \mathsf{HealthyPerson} \\
\mathsf{HealthyPerson} &\sqsubseteq \mathsf{Person}
\end{aligned}
$$

$$
\mathsf{Vegetarian}(x) \rightarrow \mathsf{Animal}(x)
$$

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\begin{aligned}
\mathsf{Vegetarian}(x) &\rightarrow \mathsf{Animal}(x) \\
\mathsf{Animal}(x) &\rightarrow \mathsf{eats}(x, f_1(x)) \\
\vdots \quad &\ \vdots \qquad \vdots \\
\mathsf{Animal}(x) &\rightarrow \mathsf{eats}(x, f_5(x)) \\
\mathsf{Animal}(x) &\rightarrow f_i(x) \not\approx f_j(x) \quad \text{for } 1 \leq i < j \leq 5
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\mathsf{Meat}(x) \wedge \mathsf{SideDish}(x) &\rightarrow \bot \\
\mathsf{Vegetarian}(x) \wedge \mathsf{eats}(x, z_1) &\rightarrow \mathsf{SideDish}(z_1)
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\mathsf{Meat}(x) \wedge \mathsf{SideDish}(x) &\to \bot\\
\mathsf{Vegetarian}(x) \wedge \mathsf{eats}(x, z_1) &\to \mathsf{SideDish}(z_1)\\
\bigwedge_{i=1}^{5} \mathsf{eats}(x, z_i) &\to \mathsf{HealthyPerson}(x) \vee \bigvee_{1 \leq i < j \leq 5} z_i \approx z_j \vee \bigvee_{i=1}^{5} \mathsf{Meat}(z_i)
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\text{Meat}(x) \wedge \text{SideDish}(x) &\rightarrow \bot\\
\text{Vegetarian}(x) \wedge \text{eats}(x, z_1) &\rightarrow \text{SideDish}(z_1)\\
\bigwedge_{i=1}^{5} \text{eats}(x, z_i) &\rightarrow \text{HealthyPerson}(x) \vee \bigvee_{1 \leq i < j \leq 5} z_i \approx z_j \vee \bigvee_{i=1}^{5} \text{Meat}(z_i)\\
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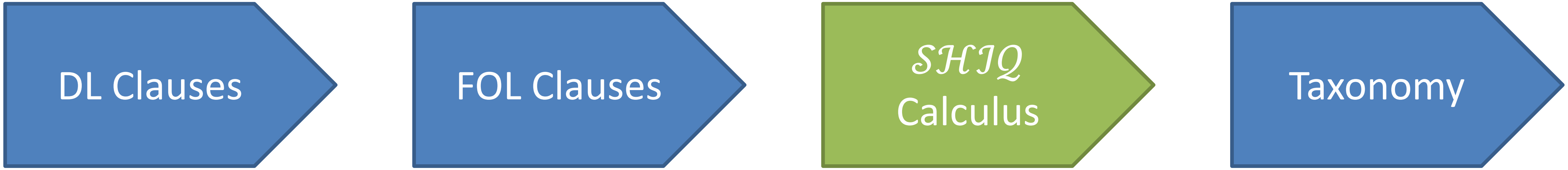
DL Clauses
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Taxonomy

Contexts

Set $\mathcal{V}$ of contexts

Each context $v \in \mathcal{V}$:

$$\boxed{A(x) \wedge B(x)} \longleftarrow \mathrm{core}_v$$

$$\left.\begin{array}{c} \mathrm{core}_v \to \cdots \\ \mathrm{core}_v \wedge C(x) \to \cdots \\ \vdots \\ \mathrm{core}_v \wedge R(y,x) \to \cdots \end{array}\right\} \mathcal{S}_v$$

Edges between contexts labelled with functions

Context structure $\mathcal{D}$ is a the graph of labelled contexts and edges

Contexts

Set $\mathcal{V}$ of contexts

Each context $v \in \mathcal{V}$:

$$\begin{array}{|c|}\hline A(x) \wedge B(x) \\ \hline \end{array} \longleftarrow \mathrm{core}_v$$

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Edges between contexts labelled with functions

Context structure $\mathcal{D}$ is a the graph of labelled contexts and edges

Sound Context Structures

$$\begin{array}{|rcl|}\hline \multicolumn{3}{|c|}{\mathrm{core}_v = A(x) \wedge B(x)} \\ \hline \top & \to & \cdots \\ \top \wedge C(x) & \to & \cdots \\ & \vdots & \\ \top \wedge R(y,x) & \to & \cdots \\ \hline \end{array} \xrightarrow{\quad f_k \quad} \begin{array}{|rcl|}\hline \multicolumn{3}{|c|}{\mathrm{core}_u = C(x) \wedge D(x)} \\ \hline \top & \to & \cdots \\ \top \wedge S(x,y) & \to & \cdots \\ & \vdots & \\ \top & \to & \cdots \\ \hline \end{array}$$

① $\mathcal{O} \vDash \mathrm{core}_v \wedge \Gamma \to \Delta$ for each $v \in V$ and each $\Gamma \to \Delta \in \mathcal{S}_v$

② $\mathcal{O} \vDash \mathrm{core}_u \to \mathrm{core}_v\{x \mapsto f_k(x), y \mapsto x\}$ for each $\langle u, v, f_k \rangle \in \mathcal{E}$

$\text{Vegetarian}(x)$
$\top \rightarrow \text{Vegetarian}(x)$

Vegetarian(x)
$\top \to \mathrm{Vegetarian}(x)$ $\top \to \mathrm{Animal}(x)$ $\top \to \mathrm{eats}(x, f_1(x))$ $\vdots$ $\top \to \mathrm{eats}(x, f_5(x))$ $\top \to f_i(x) \not\approx f_j(x)$ for $1 \leq i < j \leq 5$ $\top \to \mathrm{SideDish}(f_1(x))$ $\vdots$ $\top \to \mathrm{SideDish}(f_5(x))$

$$
\left\{
\begin{array}{rcll}
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\vdots & \vdots & \vdots & \\
\mathrm{Animal}(x) & \to & \mathrm{eats}(x, f_5(x)) & \\
\mathrm{Animal}(x) & \to & f_i(x) \not\approx f_j(x) & \text{for } 1 \leq i < j \leq 5 \\
\mathrm{Vegetarian}(x) \wedge \mathrm{eats}(x, z_1) & \to & \mathrm{SideDish}(z_1) &
\end{array}
\right\} \subseteq \mathcal{O}
$$

$$\mathrm{Vegetarian}(x)$$

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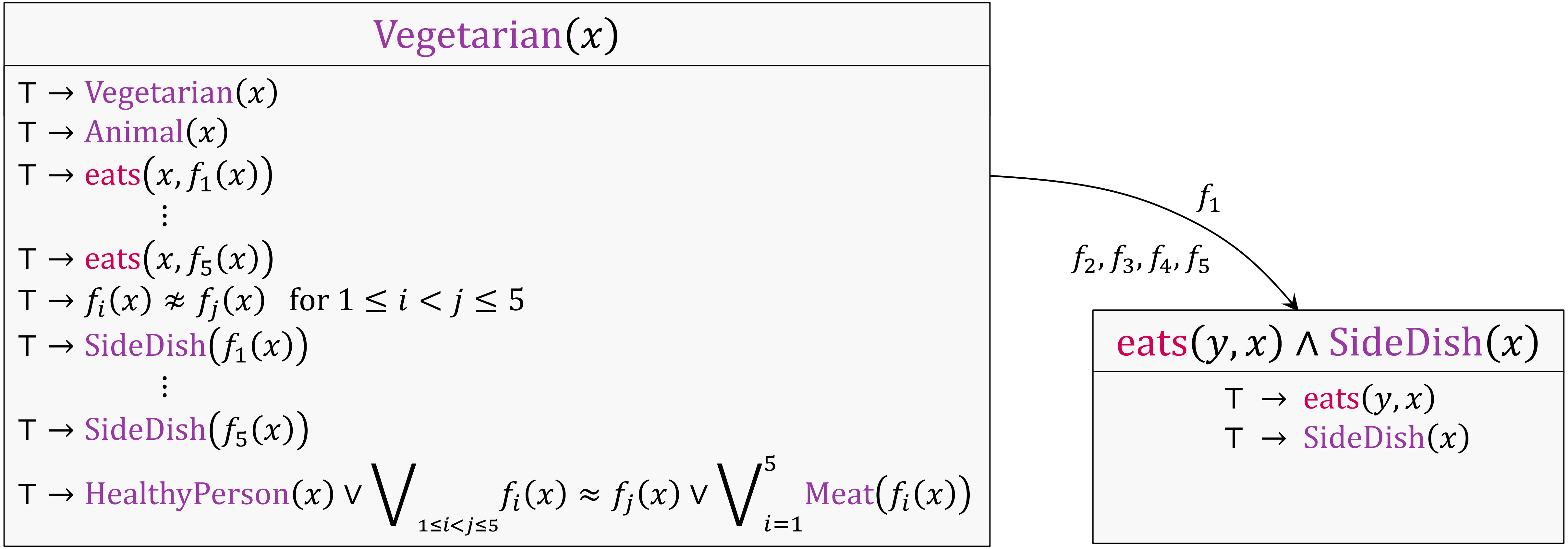
&\top \rightarrow \mathrm{eats}\big(x, f_5(x)\big)\\
&\top \rightarrow f_i(x) \not\approx f_j(x) \quad \text{for } 1 \leq i < j \leq 5\\
&\top \rightarrow \mathrm{SideDish}\big(f_1(x)\big)\\
&\qquad\qquad \vdots\\
&\top \rightarrow \mathrm{SideDish}\big(f_5(x)\big)\\
&\top \rightarrow \mathrm{HealthyPerson}(x) \vee \bigvee_{1\leq i<j\leq 5} f_i(x) \approx f_j(x) \vee \bigvee_{i=1}^{5} \mathrm{Meat}\big(f_i(x)\big)
\end{aligned}$$

$$\left\{ \ \bigwedge_{i=1}^{5} \mathrm{eats}(x, z_i) \ \rightarrow \ \mathrm{HealthyPerson}(x) \vee \bigvee_{1\leq i<j\leq 5} z_i \approx z_j \vee \bigvee_{i=1}^{5} \mathrm{Meat}(z_i) \ \right\} \subseteq \mathcal{O}$$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$\top \to \mathrm{HealthyPerson}(x) \vee \bigvee_{1\leq i<j\leq 5} f_i(x) \approx f_j(x) \vee \bigvee_{i=1}^{5} \mathrm{Meat}(f_i(x))$
f_1
f_2, f_3, f_4, f_5
$\mathrm{eats}(y, x) \wedge \mathrm{SideDish}(x)$
$\top \to \mathrm{eats}(y, x)$
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$\mathrm{Vegetarian}(x)$

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f_1

f_2, f_3, f_4, f_5

$\mathrm{eats}(y, x) \wedge \mathrm{SideDish}(x)$

$$\top \rightarrow \mathrm{eats}(y, x)$$
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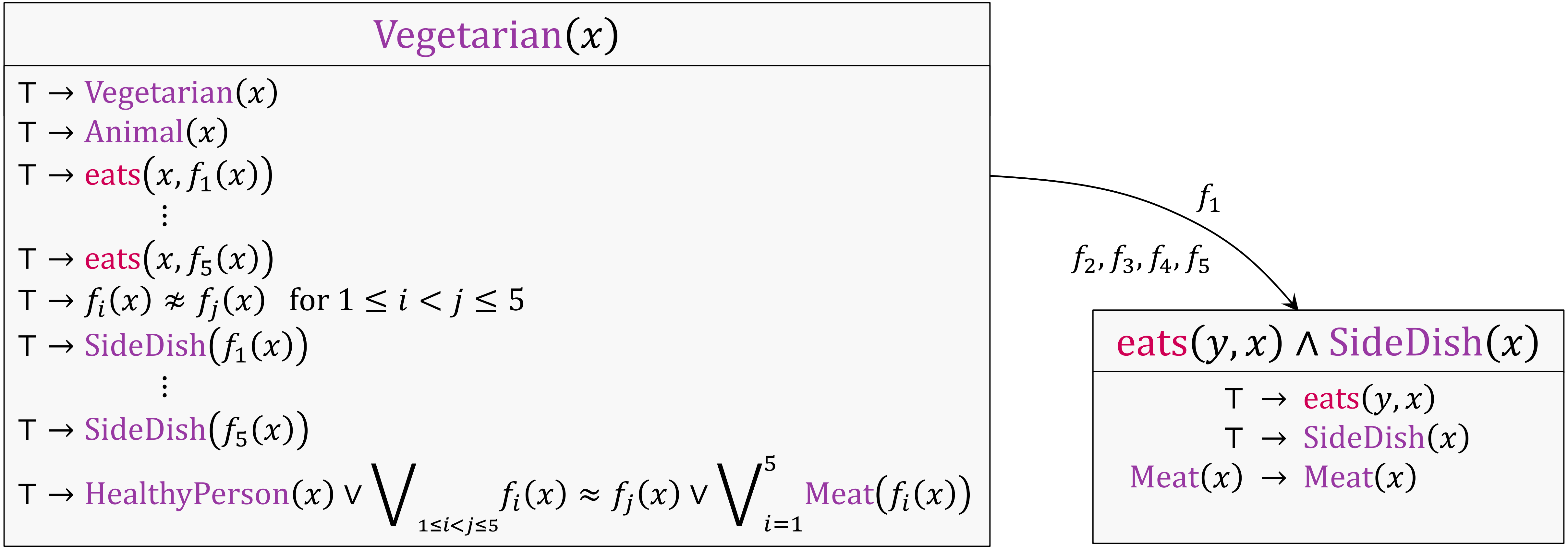
$$\left\{ \bigwedge_{i=1}^{5} \mathrm{eats}(x, z_i) \rightarrow \mathrm{HealthyPerson}(x) \vee \bigvee_{1 \leq i < j \leq 5} z_i \approx z_j \vee \bigvee_{i=1}^{5} \mathrm{Meat}(z_i) \right\} \subseteq \mathcal{O}$$



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$\mathsf{Vegetarian}(x)$

$$\top \to \mathsf{Vegetarian}(x)$$
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$$\top \to f_i(x) \not\approx f_j(x) \quad \text{for } 1 \le i < j \le 5$$
$$\top \to \mathsf{SideDish}(f_1(x))$$
$$\vdots$$
$$\top \to \mathsf{SideDish}(f_5(x))$$
$$\top \to \mathsf{HealthyPerson}(x) \lor \bigvee_{1 \le i < j \le 5} f_i(x) \approx f_j(x) \lor \bigvee_{i=1}^{5} \mathsf{Meat}(f_i(x))$$

f_1

f_2, f_3, f_4, f_5

$\mathsf{eats}(y, x) \land \mathsf{SideDish}(x)$

$$\begin{aligned} \top &\to \mathsf{eats}(y, x) \\ \top &\to \mathsf{SideDish}(x) \\ \mathsf{Meat}(x) &\to \mathsf{Meat}(x) \\ \mathsf{Meat}(x) &\to \bot \end{aligned}$$

$$\{\ \mathsf{Meat}(x) \land \mathsf{SideDish}(x) \to \bot\ \} \subseteq \mathcal{O}$$

$\mathsf{Vegetarian}(x)$

$$
\begin{aligned}
&\top \rightarrow \mathsf{Vegetarian}(x)\\
&\top \rightarrow \mathsf{Animal}(x)\\
&\top \rightarrow \mathsf{eats}\big(x, f_1(x)\big)\\
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&\top \rightarrow f_i(x) \not\approx f_j(x) \quad \text{for } 1 \leq i < j \leq 5\\
&\top \rightarrow \mathsf{SideDish}\big(f_1(x)\big)\\
&\qquad\quad\vdots\\
&\top \rightarrow \mathsf{SideDish}\big(f_5(x)\big)\\
&\top \rightarrow \mathsf{HealthyPerson}(x) \vee \bigvee_{1\leq i<j\leq 5} f_i(x) \approx f_j(x) \vee \bigvee_{i=1}^{5} \mathsf{Meat}\big(f_i(x)\big)\\
&\ldots\\
&\top \rightarrow \mathsf{HealthyPerson}(x) \vee \bigvee_{1\leq i<j\leq 5} f_i(x) \approx f_j(x)
\end{aligned}
$$

f_1

f_2, f_3, f_4, f_5

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\mathsf{Meat}(x) &\rightarrow \bot
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$\top \rightarrow \mathrm{Animal}(x)$

$\top \rightarrow \mathrm{eats}\big(x, f_1(x)\big)$

$\vdots$

$\top \rightarrow \mathrm{eats}\big(x, f_5(x)\big)$

$\top \rightarrow f_i(x) \not\approx f_j(x)$ for $1 \leq i < j \leq 5$

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$\vdots$

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$\ldots$

$\top \rightarrow \mathrm{HealthyPerson}(x) \vee \bigvee_{1\leq i<j\leq 5} f_i(x) \approx f_j(x)$

f_1

f_2, f_3, f_4, f_5

$\mathrm{eats}(y, x) \wedge \mathrm{SideDish}(x)$

$\top \rightarrow \mathrm{eats}(y, x)$

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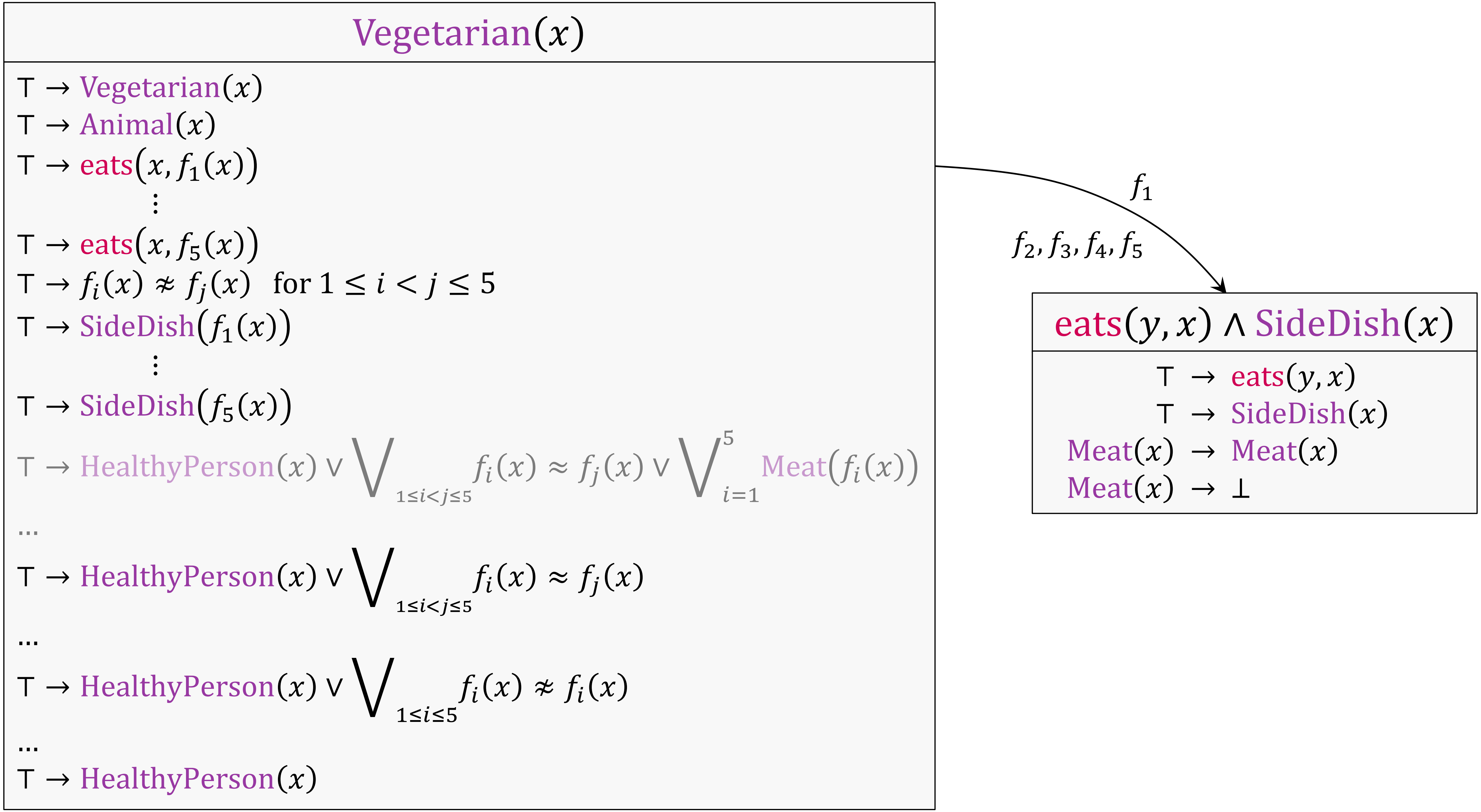
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...
$\top \rightarrow \mathrm{HealthyPerson}(x) \vee \bigvee_{1 \leq i < j \leq 5} f_i(x) \approx f_j(x)$
...
$\top \rightarrow \mathrm{HealthyPerson}(x) \vee \bigvee_{1 \leq i \leq 5} f_i(x) \not\approx f_i(x)$
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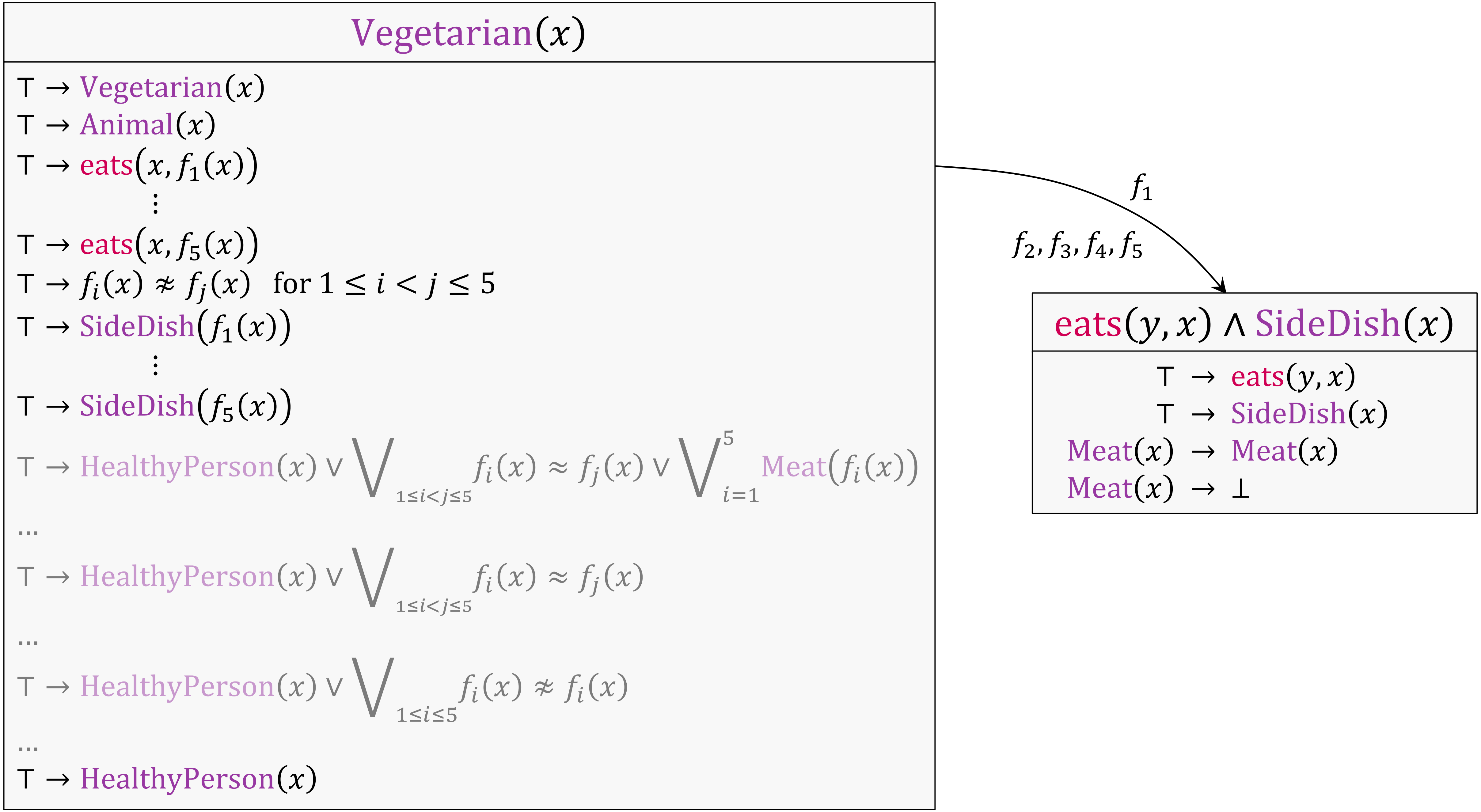
$\mathrm{Meat}(x) \rightarrow \mathrm{Meat}(x)$
$\mathrm{Meat}(x) \rightarrow \bot$



Vegetarian(x)
⊤ → Vegetarian(x)
⊤ → Animal(x)
⊤ → eats(x, f1(x))
⋮
⊤ → eats(x, f5(x))
⊤ → fi(x) ≉ fj(x) for 1 ≤ i < j ≤ 5
⊤ → SideDish(f1(x))
⋮
⊤ → SideDish(f5(x))
⊤ → HealthyPerson(x) ∨ ⋁1≤i<j≤5 fi(x) ≈ fj(x) ∨ ⋁5i=1 Meat(fi(x))
…
⊤ → HealthyPerson(x) ∨ ⋁1≤i<j≤5 fi(x) ≈ fj(x)
…
⊤ → HealthyPerson(x) ∨ ⋁1≤i≤5 fi(x) ≉ fi(x)
…
⊤ → HealthyPerson(x)
f1
f2, f3, f4, f5
eats(y, x) ∧ SideDish(x)
⊤ → eats(y, x)
⊤ → SideDish(x)
Meat(x) → Meat(x)
Meat(x) → ⊥

$$\mathsf{Vegetarian}(x)$$

$\top \to \mathsf{Vegetarian}(x)$

$\top \to \mathsf{Animal}(x)$

$\top \to \mathsf{eats}\big(x, f_1(x)\big)$

$\vdots$

$\top \to \mathsf{eats}\big(x, f_5(x)\big)$

$\top \to f_i(x) \not\approx f_j(x) \quad \text{for } 1 \le i < j \le 5$

$\top \to \mathsf{SideDish}\big(f_1(x)\big)$

$\vdots$

$\top \to \mathsf{SideDish}\big(f_5(x)\big)$

$\top \to \mathsf{HealthyPerson}(x) \vee \bigvee_{1 \le i < j \le 5} f_i(x) \approx f_j(x) \vee \bigvee_{i=1}^{5} \mathsf{Meat}\big(f_i(x)\big)$

…

$\top \to \mathsf{HealthyPerson}(x) \vee \bigvee_{1 \le i < j \le 5} f_i(x) \approx f_j(x)$

…

$\top \to \mathsf{HealthyPerson}(x) \vee \bigvee_{1 \le i \le 5} f_i(x) \not\approx f_i(x)$

…

$\top \to \mathsf{HealthyPerson}(x)$

$\top \to \mathsf{Person}(x)$

f_1

f_2, f_3, f_4, f_5

$$\mathsf{eats}(y, x) \wedge \mathsf{SideDish}(x)$$

$\top \to \mathsf{eats}(y, x)$

$\top \to \mathsf{SideDish}(x)$

$\mathsf{Meat}(x) \to \mathsf{Meat}(x)$

$\mathsf{Meat}(x) \to \bot$

$$\{\mathsf{HealthyPerson}(x) \to \mathsf{Person}(x)\} \subseteq \mathcal{O}$$

$\mathrm{Vegetarian}(x)$

$$
\begin{aligned}
&\top \to \mathrm{Vegetarian}(x) \\
&\top \to \mathrm{Animal}(x) \\
&\top \to \mathrm{eats}\big(x, f_1(x)\big) \\
&\qquad\qquad \vdots \\
&\top \to \mathrm{eats}\big(x, f_5(x)\big) \\
&\top \to f_i(x) \not\approx f_j(x) \quad \text{for } 1 \leq i < j \leq 5 \\
&\top \to \mathrm{SideDish}\big(f_1(x)\big) \\
&\qquad\qquad \vdots \\
&\top \to \mathrm{SideDish}\big(f_5(x)\big) \\
&\top \to \mathrm{HealthyPerson}(x) \vee \bigvee_{1 \leq i < j \leq 5} f_i(x) \approx f_j(x) \vee \bigvee_{i=1}^{5} \mathrm{Meat}\big(f_i(x)\big) \\
&\ldots \\
&\top \to \mathrm{HealthyPerson}(x) \vee \bigvee_{1 \leq i < j \leq 5} f_i(x) \approx f_j(x) \\
&\ldots \\
&\top \to \mathrm{HealthyPerson}(x) \vee \bigvee_{1 \leq i \leq 5} f_i(x) \not\approx f_i(x) \\
&\ldots \\
&\top \to \mathrm{HealthyPerson}(x) \\
&\top \to \mathrm{Person}(x)
\end{aligned}
$$

f_1

f_2, f_3, f_4, f_5

$\mathrm{eats}(y, x) \wedge \mathrm{SideDish}(x)$

$$
\begin{aligned}
\top &\to \mathrm{eats}(y, x) \\
\top &\to \mathrm{SideDish}(x) \\
\mathrm{Meat}(x) &\to \mathrm{Meat}(x) \\
\mathrm{Meat}(x) &\to \bot
\end{aligned}
$$

$\mathrm{Vegetarian} \sqsubseteq \mathrm{Person}$ ✓

But that's not all…

Strategies

Context overloading

Triggers to restrict rule applications

- PAYG behaviour on fragments of $\mathcal{SRIQ}$

Ordering on atoms and Skolem functions

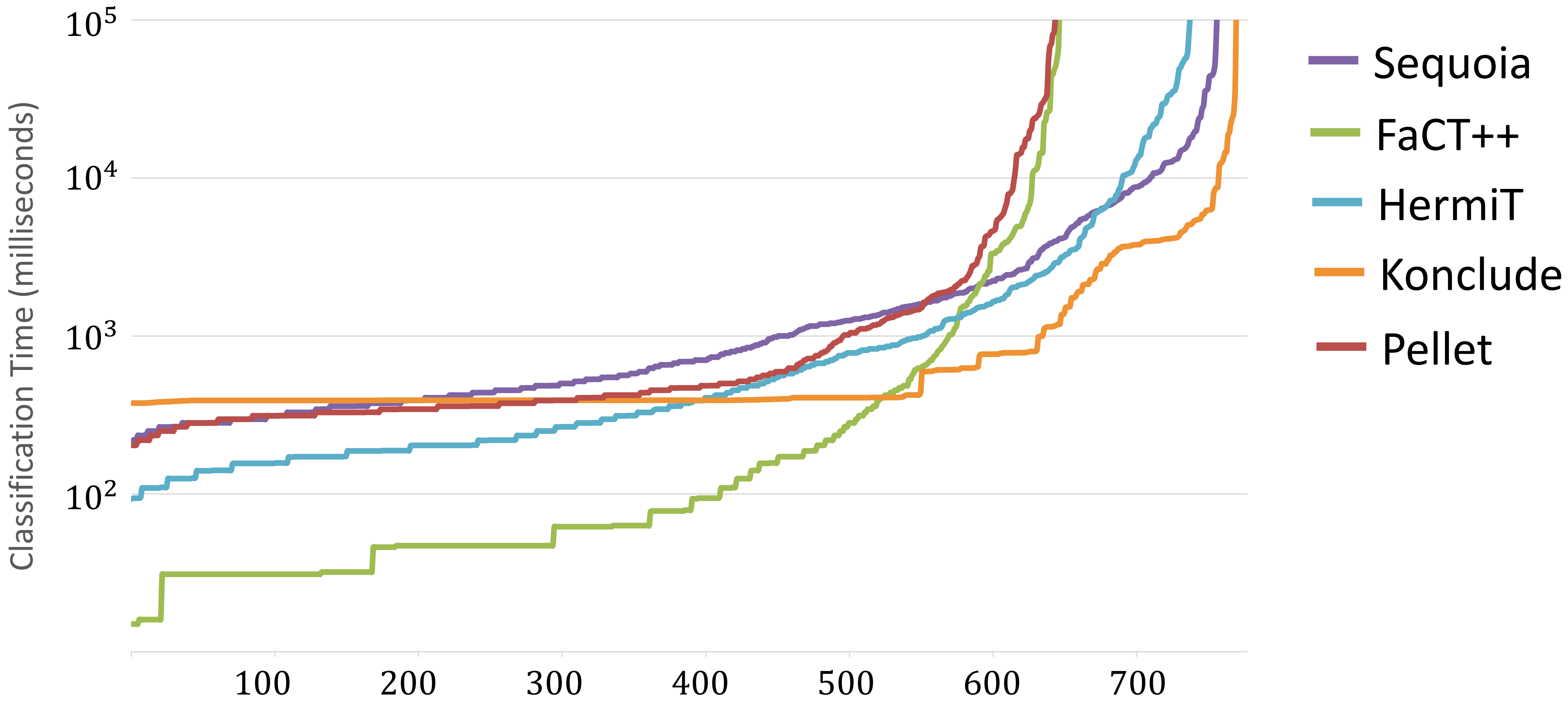
Core	If	$A \in \mathsf{core}_v$,
		and $\top \to A \notin \mathcal{S}_v$,
	then	add $\top \to A$ to $\mathcal{S}_v$.
Hyper	If	$\bigwedge_{i=1}^{n} A_i \to \Delta \in \mathcal{O}$,
		σ is a substitution such that $\sigma(x) = x$,
		$\Gamma_i \to \Delta_i \vee A_i\sigma \in \mathcal{S}_v$ with $\Delta_i \not\succeq_v A_i\sigma$ for $i \in \{1, \dots, n\}$,
		and $\bigwedge_{i=1}^{n} \Gamma_i \to \Delta\sigma \vee \bigvee_{i=1}^{n} \Delta_i \mathrel{\hat{\notin}} \mathcal{S}_v$,
	then	add $\bigwedge_{i=1}^{n} \Gamma_i \to \Delta\sigma \vee \bigvee_{i=1}^{n} \Delta_i$ to $\mathcal{S}_v$.
Eq	If	$\Gamma_1 \to \Delta_1 \vee s_1 \approx t_1 \in \mathcal{S}_v$ with $s_1 \succ_v t_1$ and $\Delta_1 \not\succeq_v s_1 \approx t_1$,
		$\Gamma_2 \to \Delta_2 \vee s_2 \circ t_2 \in \mathcal{S}_v$ with $\circ \in \{\approx, \not\approx\}$ and $s_2 \succ_v t_2$ and $\Delta_2 \not\succeq_v s_2 \circ t_2$,
		$s_2\vert_p = s_1$,
		and $\Gamma_1 \wedge \Gamma_2 \to \Delta_1 \vee \Delta_2 \vee s_2[t_1]_p \circ t_2 \mathrel{\hat{\notin}} \mathcal{S}_v$,
	then	add $\Gamma_1 \wedge \Gamma_2 \to \Delta_1 \vee \Delta_2 \vee s_2[t_1]_p \circ t_2$ to $\mathcal{S}_v$.
Ineq	If	$\Gamma \to \Delta \vee t \not\approx t \in \mathcal{S}_v$ with $\Delta \not\succeq_v t \not\approx t$,
		and $\Gamma \to \Delta \mathrel{\hat{\notin}} \mathcal{S}_v$,
	then	add $\Gamma \to \Delta$ to $\mathcal{S}_v$.
Factor	If	$\Gamma \to \Delta \vee s \approx t \vee s \approx t' \in \mathcal{S}_v$ with $\Delta \cup \{s \approx t\} \not\succeq_v s \approx t'$ and $s \succ_v t'$,
		and $\Gamma \to \Delta \vee t \not\approx t' \vee s \approx t' \mathrel{\hat{\notin}} \mathcal{S}_v$,
	then	add $\Gamma \to \Delta \vee t \not\approx t' \vee s \approx t'$ to $\mathcal{S}_v$.
Elim	If	$\Gamma \to \Delta \in \mathcal{S}_v$ and
		$\Gamma \to \Delta \mathrel{\hat{\in}} \mathcal{S}_v \setminus \{\Gamma \to \Delta\}$
	then	remove $\Gamma \to \Delta$ from $\mathcal{S}_v$.

Pred	If	$\langle u, v, f\rangle \in \mathcal{E}$,
		$\bigwedge_{i=1}^{m} A_i \to \bigvee_{i=m+1}^{m+n} A_i \in \mathcal{S}_v$,
		$\Gamma_i \to \Delta_i \vee A_i\sigma \in \mathcal{S}_u$ with $\Delta_i \not\succeq_u A_i\sigma$ for $1 \leq i \leq m$,
		$A_i \in \mathsf{Pr}(\mathcal{O})$ for each $m+1 \leq i \leq m+n$,
		and $\bigwedge_{i=1}^{m} \Gamma_i \to \bigvee_{i=1}^{m} \Delta_i \vee \bigvee_{i=m+1}^{m+n} A_i\sigma \mathrel{\hat{\notin}} \mathcal{S}_u$,
	then	add $\bigwedge_{i=1}^{m} \Gamma_i \to \bigvee_{i=1}^{m} \Delta_i \vee \bigvee_{i=m+1}^{m+n} A_i\sigma$ to $\mathcal{S}_u$;
	where	$\sigma = \{x \mapsto f(x), y \mapsto x\}$.
Succ	If	$\Gamma \to \Delta \vee A \in \mathcal{S}_u$ where $\Delta \not\succeq_u A$ and A contains $f(x)$, and
		no edge $\langle u, v, f\rangle \in \mathcal{E}$ exists such that $A \to A \mathrel{\hat{\in}} \mathcal{S}_v$ for each $A \in K_2 \setminus \mathsf{core}_v$,
	then	let $\langle v, \mathsf{core}', \succ'\rangle := \mathsf{strategy}(K_1, \mathcal{D})$;
		if $v \in \mathcal{V}$, then let $\succ_v := \succ_v \cap \succ'$, and
		otherwise let $\mathcal{V} := \mathcal{V} \cup \{v\}$, $\mathsf{core}_v := \mathsf{core}'$, $\succ_v := \succ'$, and $\mathcal{S}_v := \emptyset$;
		add the edge $\langle u, v, f\rangle$ to $\mathcal{E}$; and
		add $A \to A$ to $\mathcal{S}_v$ for each $A \in K_2 \setminus \mathsf{core}_v$;
	where	$\sigma = \{x \mapsto f(x), y \mapsto x\}$,
		$K_1 = \{\, A \in \mathsf{Su}(\mathcal{O}) \mid \top \to A\sigma \in \mathcal{S}_u \,\}$, and
		$K_2 = \{\, A \in \mathsf{Su}(\mathcal{O}) \mid \Gamma' \to \Delta' \vee A\sigma \in \mathcal{S}_u$ and $\Delta' \not\succeq_u A\sigma \,\}$.

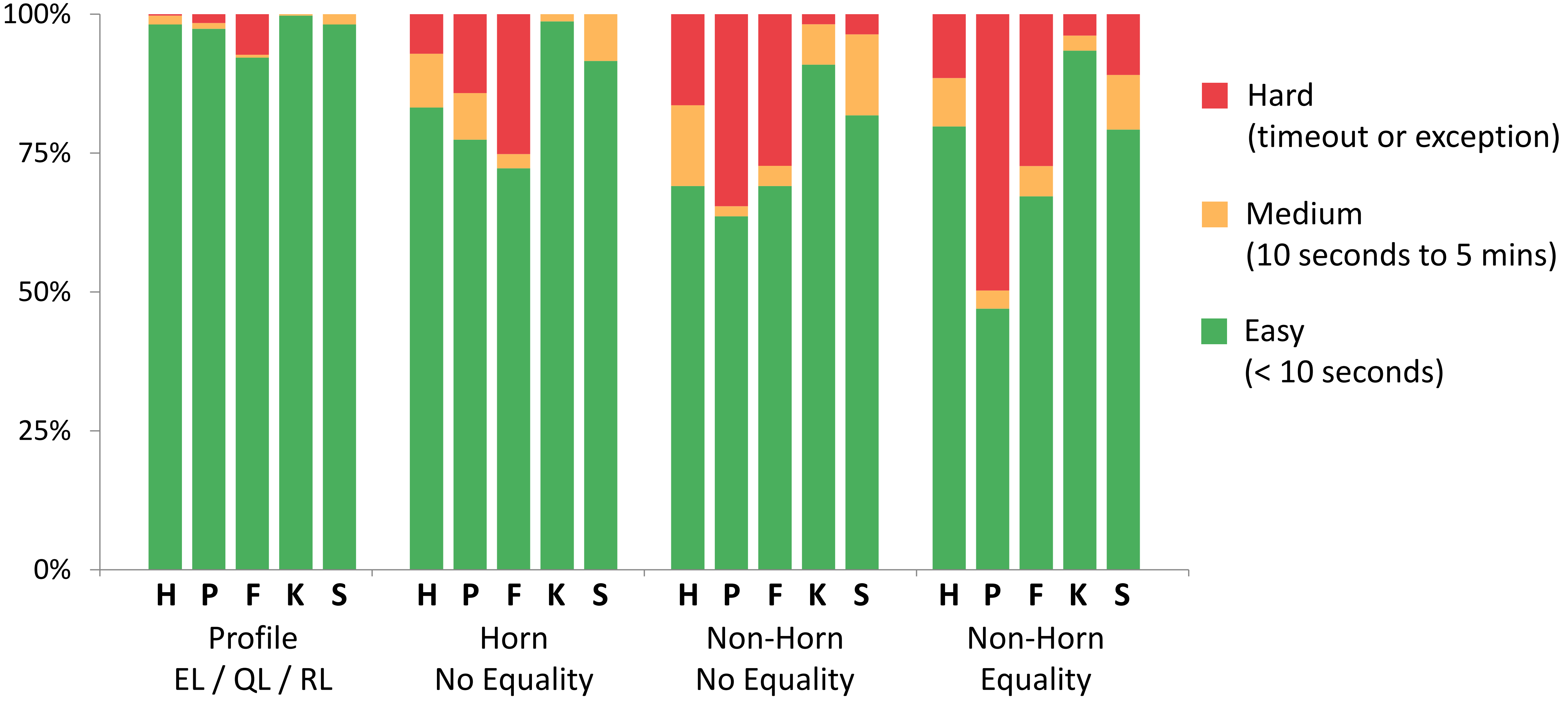
Evaluation

- Prototype implementation called *Sequoia*
- Evaluated using the Oxford Ontology Repository
  - Nominal → fresh class
  - Datatype → fresh class
  - Data property → fresh object property
  - Removed ABox assertions
- 777 ontologies
- Timeout 5 minutes
- Average over 3 runs, reporting exception or timeout as failure

Classification Times



Percentage of Easy, Medium & Hard Ontologies



Main result

Consequence-based classification for $\mathcal{SRIQ}$

Optimal worst-case complexity

Pay as you go

One pass classification

Competitive preliminary evaluation