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Dr. Burghard v. Karger

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Dear Tony,

Strangely, the two versions of the right shift axiom

$$(Q/P)\backslash R \subseteq P;Q\backslash R \cup P/(R\backslash Q) \tag{1}$$

and

$$(P/Q);R \subseteq P;Q\backslash R \cup P/(R\backslash Q) \tag{2}$$

are equivalent (modulo exchange), although there is no obvious relation between their left hand sides. I guess this is why I liked the original two-sided shift law

$$(Q/P)\backslash R \cup (P/Q);R = P;\overset{L}{Q}\backslash \overset{L}{R} \cup P/(\overset{L}{R}\backslash \overset{L}{Q}). \tag{3}$$

But I am still worried about the shift axioms

1. They are very hard to remember (or even instantiate correctly). Proof: you misquoted them in your letter.
2. I do not know if the cut axiom
$$PQ/R = P;Q/R \cup P/(R/Q) \tag{4}$$
is independent of (3).
3. In the groupoid representation, the shift axiom follows easily from confluence, but I can't show the converse.
4. I only ever used (3) once, namely to prove
$$P/L;L \subseteq PL/L \quad \text{and} \quad L;L\backslash P \subseteq L\backslash LP. \tag{5}$$

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But I can't prove the shift axiom from (5), not even with the aid of the cut law, except for sequential set algebras.

5. (5) is independent of the cut law. In a sequential set algebra, (5) corresponds precisely to observations being left and right confluent

$$\overleftarrow{x} = \overleftarrow{y} \;\Rightarrow\; \exists x', y' : x; x' = y; y' \quad \text{and the dual.} \tag{6}$$

   The cut law is needed to prove that in (5) the reverse inequalities also hold. But the shift axiom also suffices to prove equality in (5)!

There is a strong temptation to use (5) instead of (3), because it is simpler, independent of the cut axiom and sufficient in practice. But if we do so, then probably the shift laws will not be theorems of the calculus. What is your advice?

I enjoyed your note on the relative converse. The essential insight is, I think, that we should not restrict ourselves to unique compositions (for each p, q there is at most one $p; q$), but allow 'nondeterministic' composition.

By the way, this is precisely the starting point of a theory called arrow logic. There seems to exist a theory of correspondences between laws at the observation level and laws at the set level, for example in Venema's "Lecture Notes on Modal Logic". Not being a logician, I find it hard to follow the arguments.

Going through my sequential algebraic proofs of the axioms of linear temporal logic, I noticed that I used sequential calculus only in two places. In other words, essentially all the laws of LTL can be derived from just two axioms, one of which is a Galois connection, and the other a weak inverse law. I hope you will enjoy the enclosed note.

Yours sincerely,

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