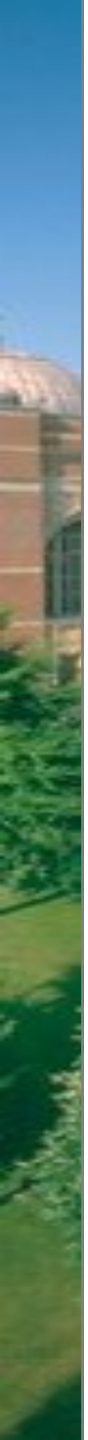

Verification and Strategy Synthesis for Stochastic Games

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Radboud University, March 2020

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Dave Parker

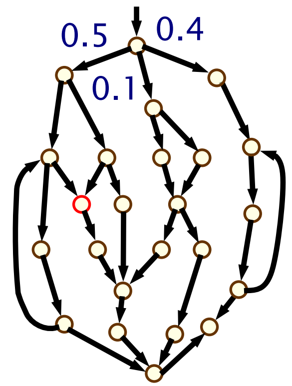
University of Birmingham

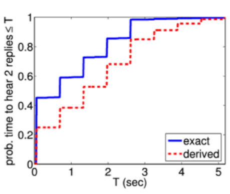
Joint work with:

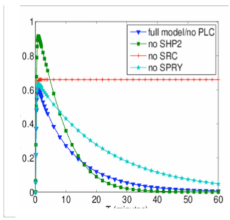
Gabriel Santos, Gethin Norman, Marta Kwiatkowska, …

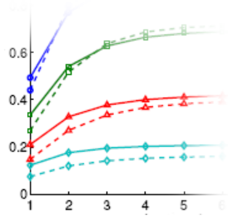
Probabilistic model checking

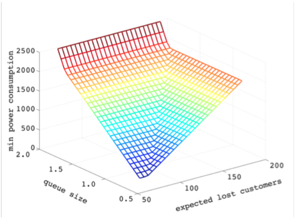
- Probabilistic model checking
  - formal construction/analysis of probabilistic models
  - "correctness" properties expressed in temporal logic
  - e.g. trigger → $P_{\geq 0.999}$ [$F^{\leq 20}$ deploy]
  - mix of exhaustive & numerical/quantitative reasoning







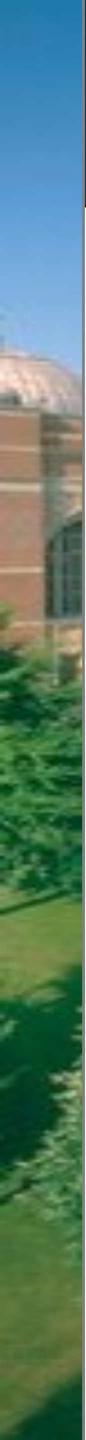




- Trends and advances
  - improvement in scalability to larger models
  - increasingly expressive/powerful model classes
  - from verification problems to control problems
  - ever widening range of application domains

Stochastic games

- Verification of systems with
  - competitive or collaborative behaviour between multiple rational agents, possibly with differing/opposing goals
  - e.g. security protocols, algorithms for distributed consensus, energy management, autonomous robotics, auctions
- Goals
  - synthesise (single or joint) strategies that are robust in adversarial settings and stochastic environments
  - analyse the effectiveness of incentive/reward schemes designed for robustness against selfish behaviour
- Natural to take a game-theoretic approach
  - we use stochastic multi-player games
  - probabilistic model checking using PRISM-games

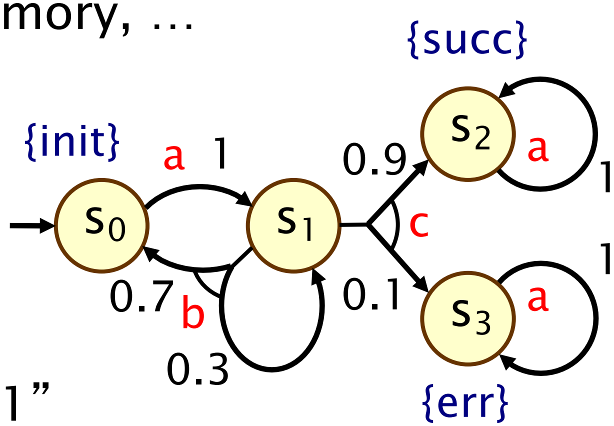


Overview

- Strategy synthesis
  - Markov decision processes (MDPs)
  - example: robot navigation
- Stochastic multi-player games (SMGs)
  - rPATL model checking and strategy synthesis
  - example: energy management
- Concurrent stochastic games (CSGs)
  - example: investor models
- Equilibria-based properties
  - (social welfare) Nash equilibria
  - example: multi-robot coordination

Verification vs. Strategy synthesis

- Markov decision processes (MDPs)
  - models nondeterministic (actions, strategies) and probabilistic behaviour
  - strategies (policies): randomisation, memory, …

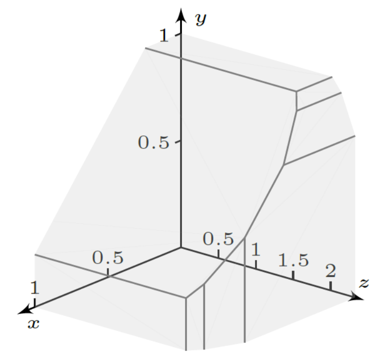


- 1. Verification
  - quantify over all possible strategies (i.e. best/worst-case)
  - $P_{\leq 0.1}$ [F *err*] : "for all strategies, the probability of an error occurring is $\leq$ 0.1"

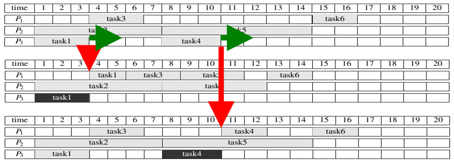
- 2. Strategy synthesis
  - generation of "correct-by-construction" controllers
  - $P_{\leq 0.1}$ [F *err*] : "does there exist a strategy for which the probability of an error occurring is $\leq$ 0.1?"

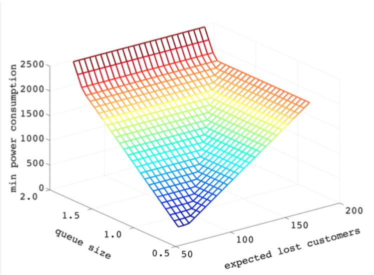
Strategy synthesis for MDPs

- Core property: probabilistic reachability
  - solvable with value iteration, policy iteration, linear programming, interval iteration, …



- Wide range of useful extensions
  - expected costs/rewards
  - linear temporal logic (LTL)
  - multi-objective model checking
  - real-time (PTAs)
  - partial observability (POMDPs)

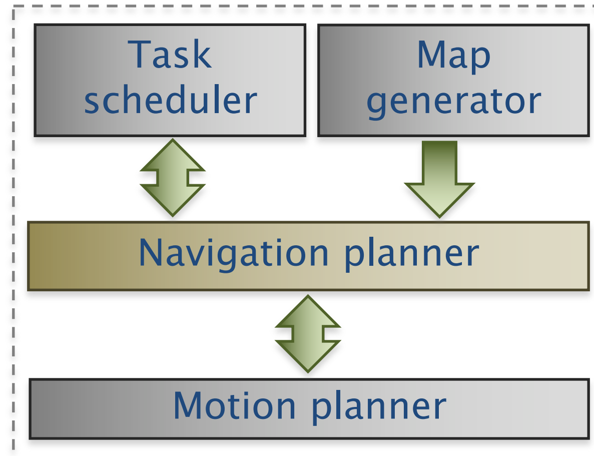


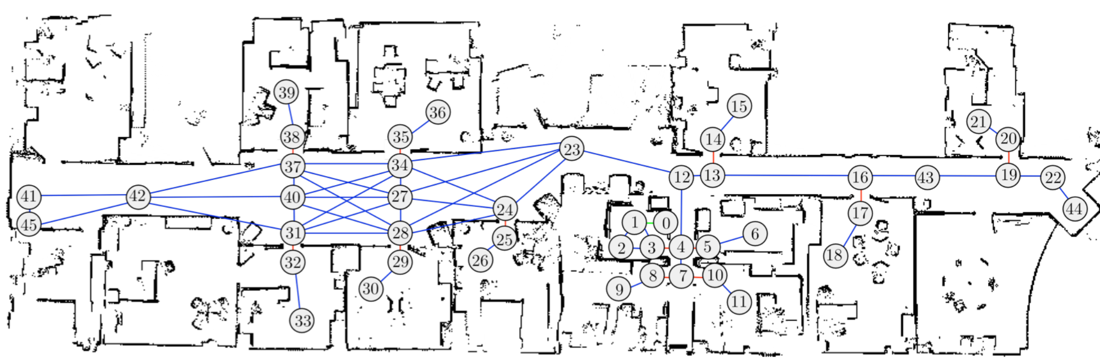


- Applications
  - dynamic power management, robot navigation, UUV mission planning, task scheduling

Application: Robot navigation

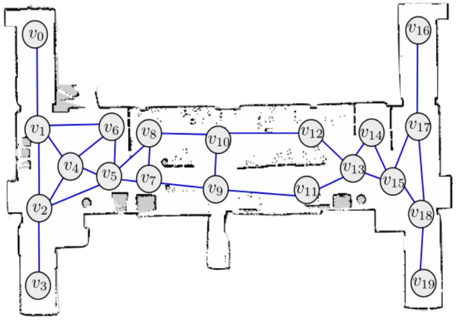
- Robot navigation planning: [IROS'14,IJCAI'15,ICAPS'17,IJRR'18]
  - learnt MDP models navigation through uncertain environment
  - co-safe LTL used to formally specify tasks to be executed by robot
  - finite-memory strategy synthesis to construct plans/controllers
  - ROS module based on PRISM
  - 100s of hrs of autonomous deployment





G4S Technology, Tewkesbury (STRANDS)

Application: Robot navigation



- Navigation planning MDPs
  - expected timed on edges + probabilities
  - learnt using data from previous explorations
- LTL-based task specification
  - expected time to satisfy (one or more) co-safe LTL formulas
  - e.g. $R_{min=?}$ [$\neg$zone$_3$ U (room$_1$ $\wedge$ (F room$_4$ $\wedge$ F room$_5$)]
- Benefits of the approach
  - LTL: flexible, unambiguous property specification
  - efficient, fully-automated techniques
  - generates meaningful guarantees on performance
    - c.f. ad-hoc reward structures, e.g. with discounting
    - QoS guarantees fed into task planning

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  - example: investor models
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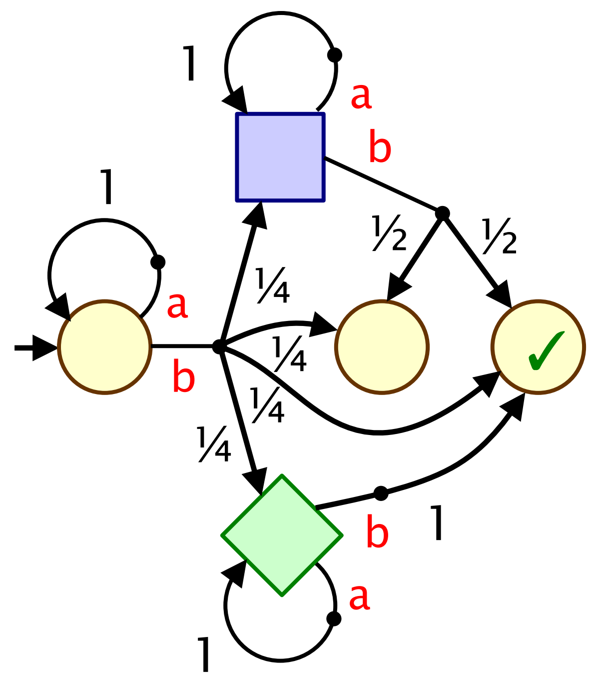
  - (social welfare) Nash equilibria
  - example: multi-robot coordination

Stochastic multi-player games

- Stochastic multi-player game (SMGs)
  - nondeterminism + probability + multiple players
  - for now: turn-based (players control states)
  - applications: e.g. security (system vs. attacker), controller synthesis (controller vs. environment)

- A (turn-based) SMG is a tuple $(N, S, \langle S_i \rangle_{i \in N}, A, \delta, L)$ where:
  - N is a set of n players
  - S is a (finite) set of states
  - $\langle S_i \rangle_{i \in N}$ is a partition of S
  - A is a set of action labels
  - $\delta : S \times A \to Dist(S)$ is a (partial) transition probability function
  - $L : S \to 2^{AP}$ is a labelling function



Strategies, probabilities & rewards

- Strategy for player i: resolves choices in S_i states
  - based on execution history, i.e. $\sigma_i : (SA)^*S_i \rightarrow Dist(A)$
  - can be: deterministic (pure), randomised, memoryless, finite-memory, …
  - Σ_i denotes the set of all strategies for player i

- Strategy profile: strategies for all players: $\sigma=(\sigma_1,\ldots,\sigma_n)$
  - probability measure Pr_s^σ over (infinite) paths from state s
  - expectation $E_s^\sigma(X)$ of random variable X over Pr_s^σ

- Rewards (or costs)
  - non-negative integers on states/transitions
  - e.g. elapsed time, energy consumption, number of packets lost, net profit, …

Property specification: rPATL

- rPATL (reward probabilistic alternating temporal logic)
  - branching-time temporal logic for SMGs
- CTL, extended with:
  - coalition operator $\langle\langle C \rangle\rangle$ of ATL
  - probabilistic operator P of PCTL
  - generalised (expected) reward operator R from PRISM
- In short:
  - zero-sum, probabilistic reachability + expected total reward
- Example:
  - $\langle\langle \{1,3\} \rangle\rangle \, P_{<0.01} [\, F^{\leq 10} \, \mathrm{error} \,]$
  - "players 1 and 3 have a strategy to ensure that the probability of an error occurring within 10 steps is less than 0.01, regardless of the strategies of other players"

rPATL syntax/semantics

- Syntax:

$$\phi ::= \mathrm{true} \mid a \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle C\rangle\rangle P_{\bowtie q}[\psi] \mid \langle\langle C\rangle\rangle R^{r}_{\bowtie x}\,[\rho]$$

$$\psi ::= X\,\phi \mid \phi\, U^{\leq k}\, \phi \mid \phi\, U\, \phi$$

$$\rho ::= I^{=k} \mid C^{\leq k} \mid F\,\phi$$

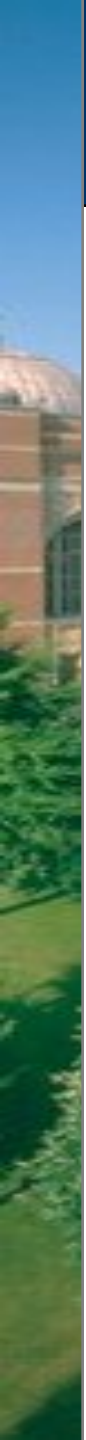
- where:
  - $a \in AP$ is an atomic proposition, $C \subseteq N$ is a coalition of players, $\bowtie \in \{\leq,<,>,\geq\}$, $q \in [0,1] \cap \mathbb{Q}$, $x \in \mathbb{Q}_{\geq 0}$, $k \in \mathbb{N}$

    r is a reward structure

- Semantics:
- e.g. P operator: $s \vDash \langle\langle C\rangle\rangle P_{\bowtie q}[\psi]$ iff:
  - "there exist strategies for players in coalition C such that, for all strategies of the other players, the probability of path formula ψ being true from state s satisfies ⋈ q"

rPATL and beyond

- Quantitative (numerical) properties:
  - $\langle\langle\{1\}\rangle\rangle\ P_{max=?}[\ F\ error\ ]$, i.e. $\sup_{\sigma_1\in\Sigma_1} \inf_{\sigma_2\in\Sigma_2} Pr_s^{\sigma_1,\sigma_2}(F\ error)$
  - "what is the maximum probability of reaching an error state that player 1 can guarantee?" (against player 2)
- Nesting (and n>2 players)
  - players: $sensor_1$, $sensor_2$, repairer
  - $\langle\langle sensor_1\rangle\rangle\ P_{<0.01}[\ F\ (\neg\langle\langle repairer\rangle\rangle\ P_{\geq 0.95}[\ F\ \text{"operational"}\ ]\ )\ ]$
- Generalised reward operators [TACAS'12, FMSD'13]
  - $\langle\langle C\rangle\rangle R^r_{\bowtie x}\ [F^\star \phi]$ where $\star \in \{\infty,c,0\}$
  - F^0 is tricky: needs finite-memory strategies
- And more…
  - rPATL*, reward-bounded [FMSD], exact bounds [CONCUR'12]
  - multi-objective model checking [QEST'13,TACAS15,I&C'17]



Model checking rPATL

- Main task: checking individual P and R operators
  - reduction to solution of zero-sum stochastic 2-player game
  - (probabilistic reachability + expected total reward)
  - e.g. $\langle\langle C\rangle\rangle P_{\geq q}[\psi] \;\Leftrightarrow\; \sup_{\sigma_1 \in \Sigma_1} \inf_{\sigma_2 \in \Sigma_2} \mathrm{Pr}_s^{\sigma_1,\sigma_2}(\psi) \geq q$
  - complexity: NP ∩ coNP (without any $R[F^0]$ operators)
  - complexity for full logic: NEXP ∩ coNEXP (due to $R[F^0]$ op.)

- In practice though:
  - (usual approach taken in probabilistic model checking tools)
  - value iteration (evaluation of numerical fixed points)
  - and more: graph-algorithms, sequences of fixed points, …

Example: Probabilistic reachability

- E.g. $\langle\langle C\rangle\rangle P_{\geq q}[\ F\ \phi\ ]$: max/min reachability probabilities
  - compute $\sup_{\sigma_1 \in \Sigma_1} \inf_{\sigma_2 \in \Sigma_2} Pr_s^{\sigma_1,\sigma_2}(F\ \phi)$ for all states s
  - deterministic memoryless strategies suffice

- Value $p(s)$ for state s is least fixed point of:

$$p(s) = \begin{cases} 1 & \text{if } s \in Sat(\phi) \\ \max_{a \in A(s)} \Sigma_{s' \in S}\ \delta(s,a)(s') \cdot p(s') & \text{if } s \in S_1 \backslash Sat(\phi) \\ \min_{a \in A(s)} \Sigma_{s' \in S}\ \delta(s,a)(s') \cdot p(s') & \text{if } s \in S_2 \backslash Sat(\phi) \end{cases}$$

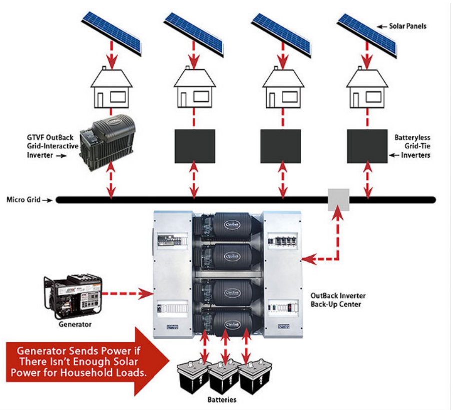
- Computation (value iteration):
  - start from zero, propagate probabilities backwards
  - guaranteed convergence; apply “usual” termination criteria

PRISM-games

- PRISM-games: www.prismmodelchecker.org/games
  - extension of PRISM modelling language (see later)
  - implementation in explicit engine
  - prototype symbolic (MTBDD) version also available
- Example application domains
  - security: attack-defence trees; DNS bandwidth amplification
  - self-adaptive software architectures
  - autonomous urban driving
  - human-in-the-loop UAV mission planning
  - collective decision making and team formation protocols
  - energy management protocols

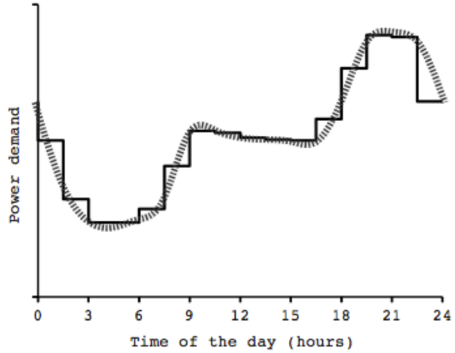
Application: Energy management

- Energy management protocol for Microgrid
  - randomised demand management protocol
  - random back-off when demand is high



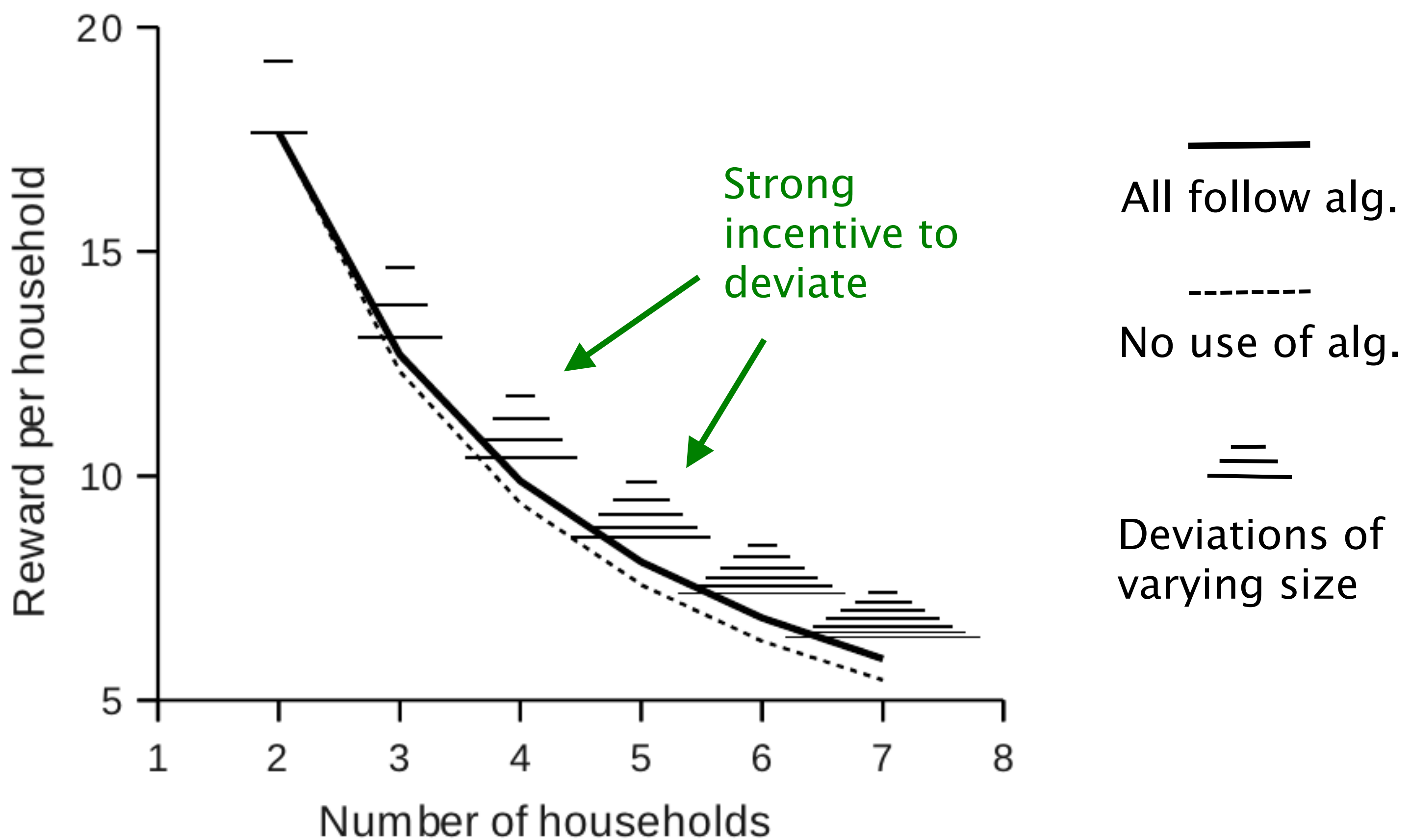
- Original analysis [Hildmann/Saffre'11]
  - protocol increases "value" for clients
  - simulation-based, clients are honest

- Our analysis
  - stochastic multi-player game model
  - clients can cheat (and cooperate)
  - model checking: PRISM-games
  - exposes protocol weakness (incentive for clients to act selfishly
  - propose/verify simple fix using penalties



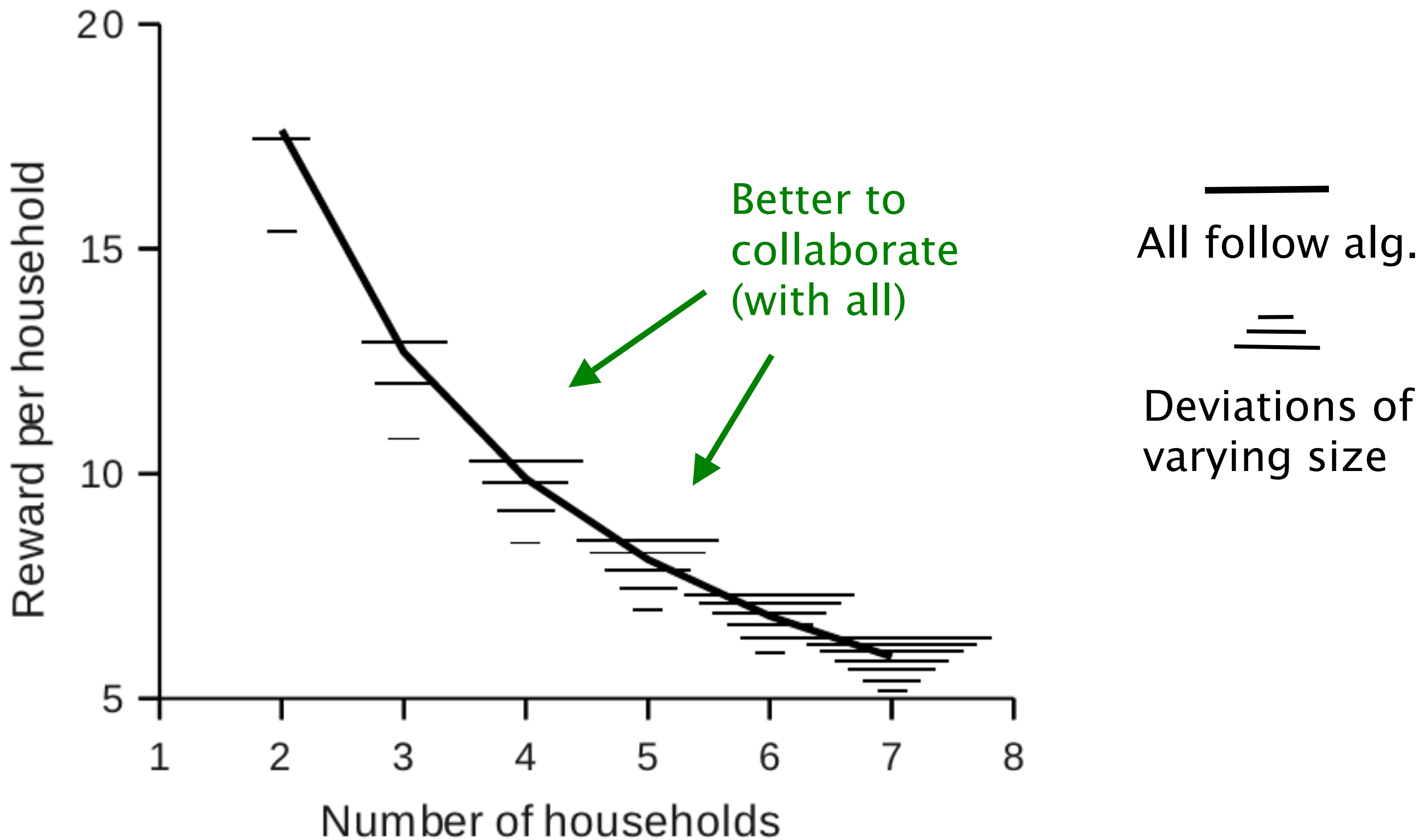
Results: Competitive behaviour

- Expected total value V per household
  - in rPATL: $\langle\langle C\rangle\rangle R^{r_C}_{max=?}\,[F^0\ time{=}max\ time]\ /\ |C|$
  - where r_C is combined rewards for coalition C



Results: Competitive behaviour

- Algorithm fix: simple punishment mechanism
  - distribution manager can cancel some loads exceeding c_{lim}



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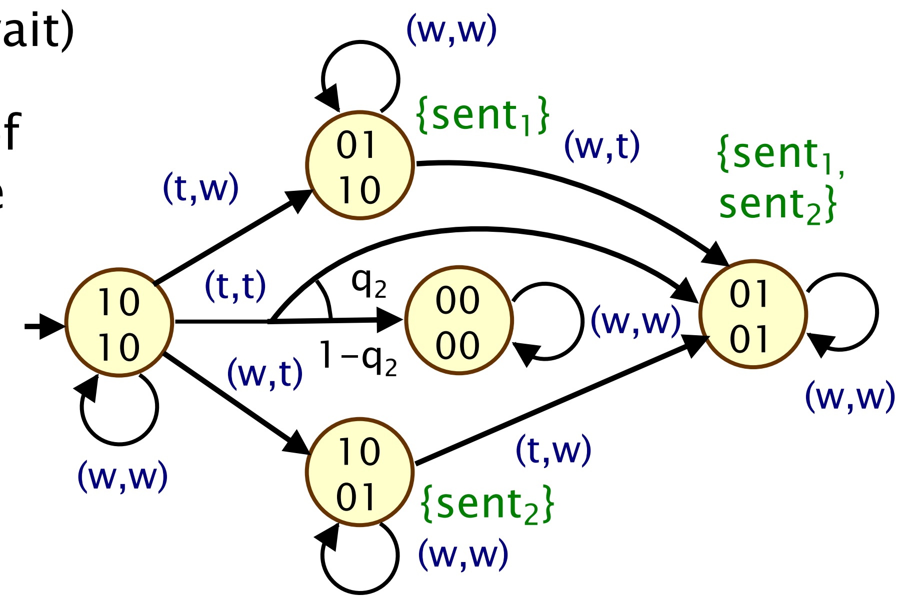
Concurrent stochastic games

- Concurrent stochastic games (CSGs)
  - players choose actions concurrently
  - jointly determines (probabilistic) successor state
  - generalises turn-based stochastic games
- Key motivation:
  - more realistic model of components operating concurrently, making action choices without knowledge of others
- Formally
  - set of n players N, state space S, actions A_i for player i
  - transition probability function $\delta : S \times A \rightarrow Dist(S)$
  - where $A = (A_1 \cup \{\perp\}) \times \ldots \times (A_n \cup \{\perp\})$
  - strategies $\sigma_i : FPath \rightarrow Dist(A_i)$, strategy profiles $\sigma=(\sigma_1,\ldots,\sigma_n)$
  - probability measure Pr_s^{σ}, expectations $E_s^{\sigma}(X)$

Example CSG: medium access control

- Example CSG: medium access control
  - 2 players (senders on a shared channel)
  - CSG states: $e_1 s_1$ $e_2 s_2$ ($energy_1/sent_1$, $energy_2/sent_2$)
  - actions = t (transmit), w (wait)
  - transmission costs 1 unit of energy and is only possible if energy is positive
  - q_2 = probability of transmission success if 2 messages sent simultaneously



(probabilistic extension of [Brenguier'13])

rPATL for CSGs

- We can use the same logic rPATL as for SMGs
- Examples for medium access control game:
  - $\langle\langle 1 \rangle\rangle \, P_{\geq 1} [\, F \; sent_1 \,]$ – can player 1 ensure that it eventually transmits with probability 1?
  - $\langle\langle 1 \rangle\rangle \, P_{max=?} [\, \neg sent_2 \; U \; sent_1 \,]$ – what is the maximum probability user 1 can ensure of being the first to transmit, regardless of the behaviour of user 2?

rPATL model checking for CSGs

- Same overall model checking algorithm [QEST'18]
  - key ingredients are solution of (zero-sum) 2-player CSGs
- E.g. $\langle\langle C\rangle\rangle P_{\geq q}[\ F\ \phi\ ]$: max/min reachability probabilities
  - compute $\sup_{\sigma_1 \in \Sigma_1} \inf_{\sigma_2 \in \Sigma_2} Pr_s^{\sigma_1,\sigma_2}(F\ \phi)$ for all states s
  - note that optimal strategies are now randomised
  - solution of the 2-player CSG is in PSPACE
  - we again use a value iteration based approach
- Value $p(s)$ for state s is least fixed point of:

$$p(s) = \begin{cases} 1 & \text{if } s \in Sat(\phi) \\ val(Z) & \text{if } s \in S \backslash Sat(\phi) \end{cases} \quad \text{where:}$$

  - Z is the matrix game with $z_{ij} = \Sigma_{s' \in S}\ \delta(s,(a_i,b_j))(s') \cdot p(s')$
  - so each iteration requires solution of a matrix game for each state (LP problem of size $|A|$, where A = action set)

Matrix games

- Matrix games
  - finite, one-shot, 2-player, zero-sum games
  - utility function $u_i : A_1 \times A_2 \to \mathbb{R}$ for each player i
  - represented by matrix Z where $z_{ij} = u_1(a_i,b_j) = -u_2(a_i,b_j)$
- Example: rock-paper-scissors
  - rock > scissors, paper > rock, scissors > paper, otherwise draw

$$Z = \begin{array}{c} \\ r \\ p \\ s \end{array} \begin{array}{c} \begin{array}{ccc} r & p & s \end{array} \\ \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \end{array}$$

- Optimal (player 1) strategy via LP solution (minimax):
  - compute value $val(Z)$: maximise value v subject to:
  - $v \leq x_p - x_s$
    $v \leq x_s - x_r,$
    $v \leq x_s - x_p$
    $x_r + x_p + x_s = 1$
    $x_r \geq 0, x_p \geq 0, x_s \geq 0$

Optimal strategy (randomised):
$(x_r, x_p, x_s) = (⅓, ⅓, ⅓)$

CSGs in PRISM-games

- CSG model checking implemented in PRISM-games 3.0
- Extension of PRISM modelling language
  - (see next slide)
- Explicit engine implementation
  - plus LPsolve library for matrix games LP solution
  - this is the main bottleneck
  - experiments with CSGs up to ~3 million states
- Case studies:
  - future markets investor, trust models for user-centric networks, intrusion detection policies, jamming radio systems

CSGs in PRISM-games 3.0

Extended version of medium access control example

```
csg
player p1 user1 endplayer
player p2 user2 endplayer
// Users (senders)
module user1
    s1 : [0..1] init 0; // has player 1 sent?
    e1 : [0..emax] init emax; // energy level of player 1
    [w1] true -> (s1'=0); // wait
    [t1]  e1>0 -> (s1'=c' ? 0 : 1) & (e1'=e1-1); // transmit
endmodule
module user2 = user1 [ s1=s2, e1=e2, w1=w2, t1=t2 ] endmodule
// Channel: used to compute joint probability distribution for transmission failure
module channel
    c : bool init false; // is there a collision?
    [t1,w2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 1 transmits
    [w1,t2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 2 transmits
    [t1,t2]  true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```

CSGs in PRISM-games 3.0

```
csg
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module user1
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endmodule
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    [t1,t2] true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```

Each player comprises one or more modules

Players have distinct actions, executed simultaneously

CSGs in PRISM-games 3.0

```
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endmodule
module user2 = user1 [ s1=s2, e1=e2, w1=w2, t1=t2 ] endmodule
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        [t1,t2]  true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```

Additional (deterministic) modules not attached to any player

CSGs in PRISM-games 3.0

```
csg
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player p2 user2 endplayer
// Users (senders)
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	s1 : [0..1] init 0; // has player 1 sent?
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endmodule
module user2 = user1 [ s1=s2, e1=e2, w1=w2, t1=t2 ] endmodule
// Channel: used to compute joint probability distribution for transmission failure
module channel
	c : bool init false; // is there a collision?
	[t1,w2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 1 transmits
	[w1,t2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 2 transmits
	[t1,t2]  true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```
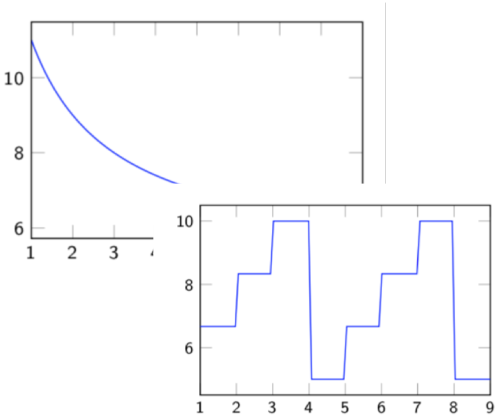

Variable updates can refer to other variables updated simultaneously

Action lists used to specify synchronisation

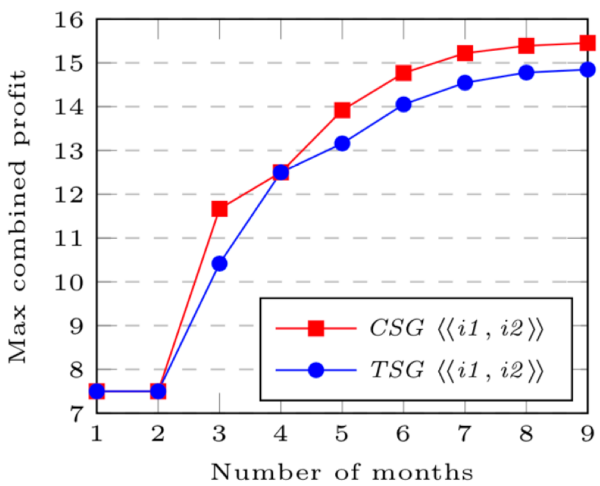
Application: Future markets investor

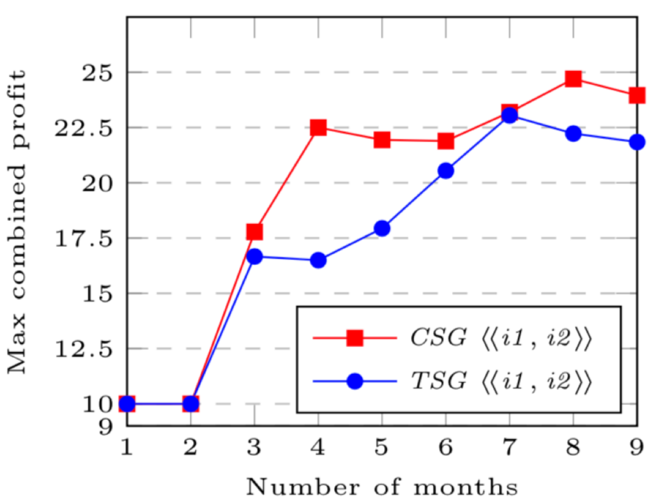


- Model of interactions between:
  - stock market, evolves stochastically
  - two investors i_1, i_2 decide when to invest
  - market decides whether to bar investors
- Modelled as a 3-player CSG
  - extends simpler model originally from [McIver/Morgan'07]
  - investing/barring decisions are simultaneous
  - profit reduced for simultaneous investments
  - market cannot observe investors' decisions
- Analysed with rPATL model checking & strategy synthesis
  - distinct profit models considered: 'normal market', 'later cash-ins' and 'later cash-ins with fluctuation'
  - comparison between TSG and CSG models

Application: Future markets investor

- Example rPATL query:
  - $\langle\langle \text{investor}_1,\text{investor}_2 \rangle\rangle\ R^{\text{profit}_{1,2}}_{\max=?}\ [\ F\ \text{finished}_{1,2}\ ]$
  - i.e. maximising joint profit
- Results: with (left) and without (right) fluctuations
  - optimal (randomised) investment strategies synthesised
  - CSG yields more realistic results (market has less power due to limited observation of investor strategies)





Overview

- Strategy synthesis
  - Markov decision processes (MDPs)
  - example: robot navigation
- Stochastic multi-player games (SMGs)
  - rPATL model checking and strategy synthesis
  - example: energy management
- Concurrent stochastic games (CSGs)
  - example: investor models
- Equilibria-based properties
  - (social welfare) Nash equilibria
  - example: multi-robot coordination

Nash equilibria

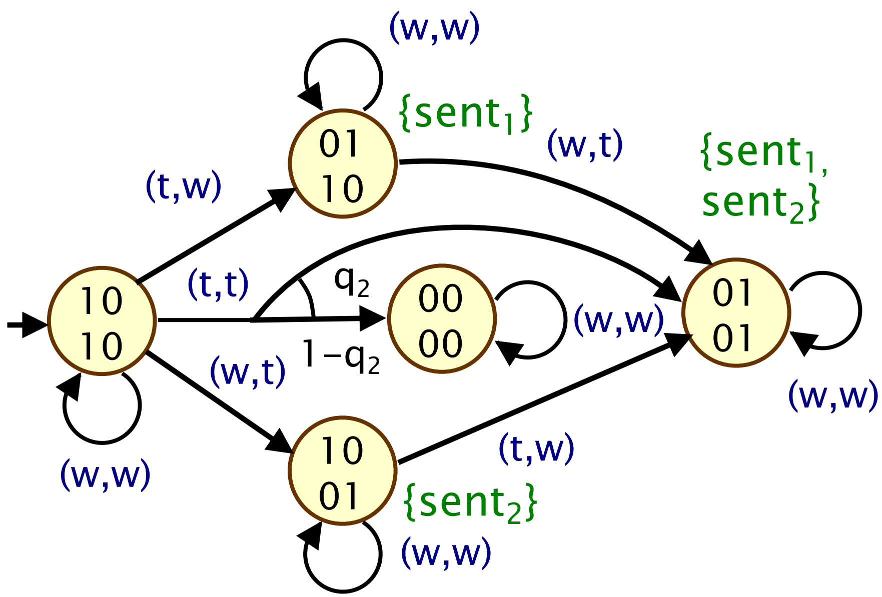
- Now consider distinct objectives X_i for each player i
  - no longer restricted to zero sum goals

- Nash equilibria (NE)
  - no incentive for any player to unilaterally change strategy
  - a strategy profile $\sigma=(\sigma_{1,\ldots,}\sigma_n)$ for a CSG is an ϵ–Nash equilibrium for state s and objectives $X_1,\ldots,X_n$ iff:
  - $E_s^{\sigma}(X_i) \geq \sup \{ E_s^{\sigma'}(X_i) \mid \sigma'=\sigma_{-i}[\sigma_i'] \text{ and } \sigma_i' \in \Sigma_i \} - \epsilon$ for all i
  - ϵ–NE (but not 0–NE) guaranteed to exist for CSGs

- Social welfare Nash equilibria (SWNE)
  - NE which maximise sum $E_s^{\sigma}(X_1) + \ldots E_s^{\sigma}(X_n)$
  - i.e., optimise combined goal

Example

- Example CSG: medium access control



- If objective X_i = probability for user i to send successfully:
  - 2 SWNEs when one user waits for the other to transmit and then transmits
- If objective X_i =probability of user i being *first* to transmit:
  - only 1 SWNE: both immediately try to transmit

rPATL + Nash operator

- Extension of rPATL for Nash equilibria [FM'19]

$$\phi ::= \text{true} \mid a \mid \neg\phi \mid \phi \wedge \phi \mid$$
$$\langle\langle C\rangle\rangle P_{\bowtie q}[\psi] \mid \langle\langle C\rangle\rangle R^r_{\bowtie x}\,[\rho] \mid \langle\langle C{:}C'\rangle\rangle_{\max\bowtie x}\,[\theta]$$

$$\theta ::= P[\psi]+P[\psi] \mid R^r[\rho]+R^r[\rho]$$

$$\psi ::= X\,\phi \mid \phi\ U^{\leq k}\ \phi \mid \phi\ U\ \phi$$

$$\rho ::= I^{=k} \mid C^{\leq k} \mid F\,\phi$$

- where:
  - $a\in AP$ is an atomic proposition, $C\subseteq N$ is a coalition of players and $C'=N\backslash C$, $\bowtie\ \in \{\leq,<,>,\geq\}$, $q \in [0,1]\cap\mathbb{Q}$, $x \in \mathbb{Q}_{\geq 0}$, $k \in \mathbb{N}$

    r is a reward structure

- Semantics:
  - $\langle\langle C{:}C'\rangle\rangle_{\max\bowtie x}\,[\theta]$ is satisfied if there exist strategies for all players that form a SWNE between coalitions C and $C'(=N\backslash C)$, and under which the *sum* of the two objectives in θ is $\bowtie x$

Model checking for extended rPATL

- Key ingredient is now:
  - solution of SWNEs for bimatrix games
  - (basic problem is EXPTIME)
  - we adapt known approach using labelled polytopes, and implement using an encoding to SMT

- Two types of model checking operator
  - bounded: backwards induction
  - unbounded: value iteration, e.g.:

$$\mathtt{V}_{\mathsf{G}^C}(s,\theta,n) = \begin{cases} (1,1) & \text{if } s \in Sat(\phi^1) \cap Sat(\phi^2) \\ (1, \mathtt{P}^{\max}_{\mathsf{G},s}(\mathtt{F}\ \phi^2)) & \text{else if } s \in Sat(\phi^1) \\ (\mathtt{P}^{\max}_{\mathsf{G},s}(\mathtt{F}\ \phi^1), 1) & \text{else if } s \in Sat(\phi^2) \\ (0,0) & \text{else if } n{=}0 \\ val(\mathsf{Z}_1, \mathsf{Z}_2) & \text{otherwise} \end{cases}$$

  - where Z_1 and Z_2 encode matrix games similar to before

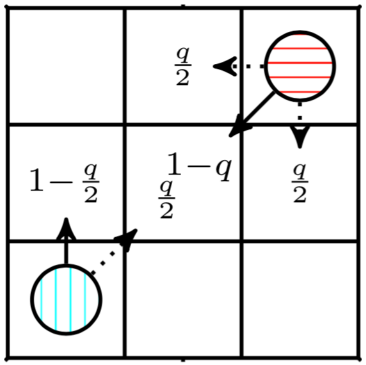
PRISM-games support

- Implementation in PRISM-games
  - extends CSG rPATL model checking implementation
  - bimatrix games solved using Z3/Yices encoding
  - optimised filtering of dominated strategies
  - scales up to CSGs with ~2 million states

- Applications
  - robot navigation in a grid, medium access control, Aloha communication protocol, power control
  - SWNE strategies outperform those found with rPATL
  - ϵ-Nash equilibria found typically have ϵ=0

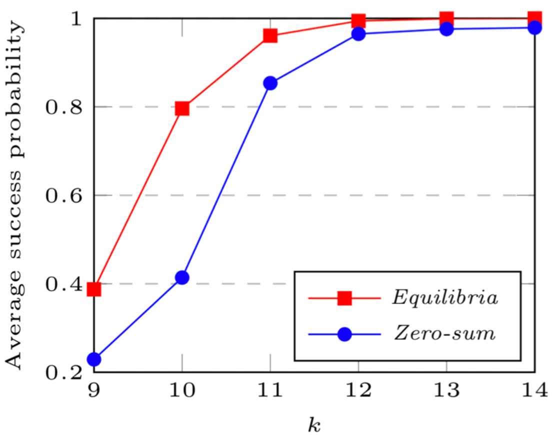
Example: multi-robot coordination

- 2 robots navigating an l x l grid
  - start at opposite corners, goals are to navigate to opposite corners
  - obstacles modelled stochastically: navigation in chosen direction fails with probability q



- We synthesise SWNEs to maximise the average probability of robots reaching their goals within time k
  - $\langle\langle robot1{:}robot2 \rangle\rangle_{max=?}$ (P [$F^{\leq k}$ $goal_1$]+P [$F^{\leq k}$ $goal_2$])

- Results (10 x 10 grid)
  - better performance obtained than using zero-sum methods, i.e., optimising for robot 1, then robot 2



Conclusions

- Probabilistic model checking & PRISM
  - verification & strategy synthesis
- Stochastic multi-player games
  - competitive/collaborative behaviour + stochasticity
  - rPATL model checking & strategy synthesis
  - concurrent stochastic games: more realistic models of competing stochastic components
  - Nash equilibria: beyond zero sum properties
- Challenges & directions
  - partial information/observability & greater efficiency
  - scalability, e.g. symbolic methods, abstraction
  - managing model uncertainty + integration with learning