Quantitative Verification: Correctness, Reliability and Beyond

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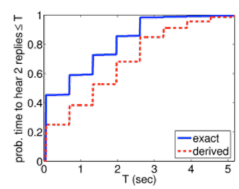
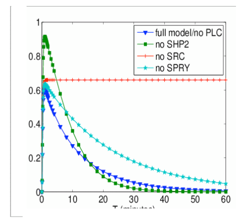
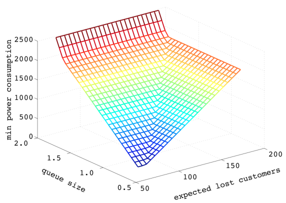
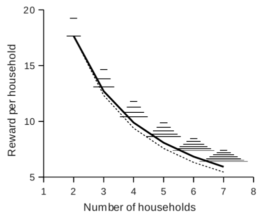
Outline

- Verification and model checking
- Quantitative verification
- Probabilistic model checking and PRISM
    - Discrete time Markov chains
    - Adding continuous-time…
        - continuous-time Markov chains
    - Adding nondeterminism…
        - Markov decision processes
    - Adding game theory…
        - stochastic multi-player games



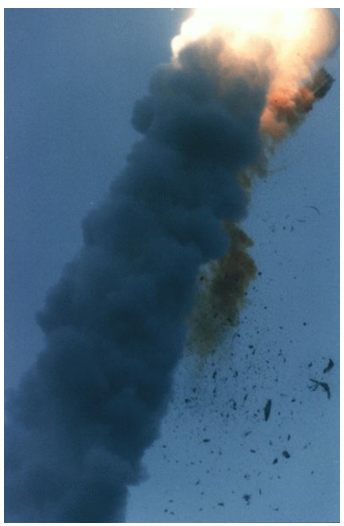






Verification

- Checking the correctness of (computerised) systems using rigorous, mathematically-sound techniques
  - in essence: proving that a piece of software, or hardware, or a protocol behaves correctly



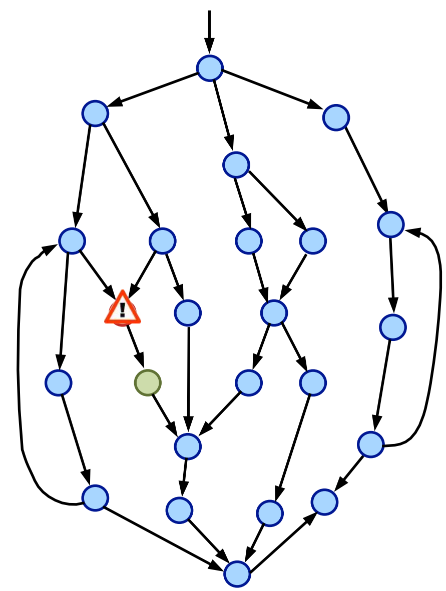
Ariane 5, flight 501

Toyota Prius

Infusion pumps

Model checking

- Automated verification: model checking
  - exhaustive construction/analysis of finite-state model
  - correctness properties expressed in temporal logic
- Successful in practice
  - e.g. Windows device driver development
  - example property: "acquire/release of spinlock always strictly alternate"



A [G (*trigger*→ X *deploy*)]

- Why it works
  - temporal logic: expressive, tractable
  - fully automated, tools available
  - not just verification, but falsification, i.e. bug hunting

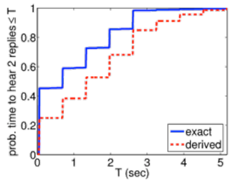
Quantitative verification

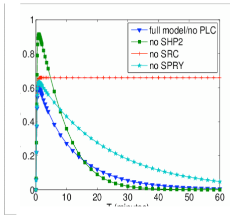
- Adds quantitative aspects (to models and properties)
  - probability, time, costs, rewards, …
- Probability
  - physical components can fail
  - communication media are unreliable
  - algorithms/protocols use randomisation

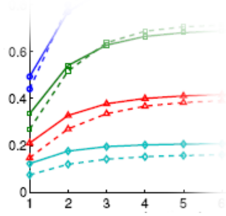
- Time
  - delays, time-outs, failure rates, …
- Costs & rewards
  - energy consumption, resource usage, …
  - profit, incentive schemes, …

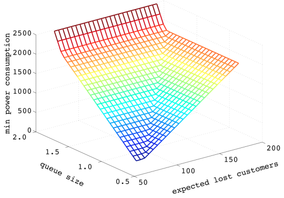
Quantitative verification

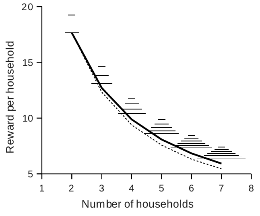
- Correctness properties are quantitative
  - "the probability of an airbag failing to deploy within 0.02 seconds of being triggered is at most 0.001"
  - "with probability 0.99, the packet arrives within 10 ms"
- Beyond correctness:
  - reliability, timeliness, performance, efficiency, …
  - "the expected energy consumption of the sensor"
  - "the expected number of FGF ligands after 20 minutes"







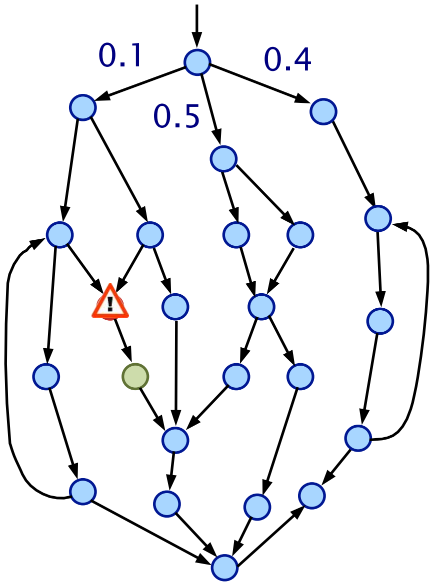




Probabilistic model checking

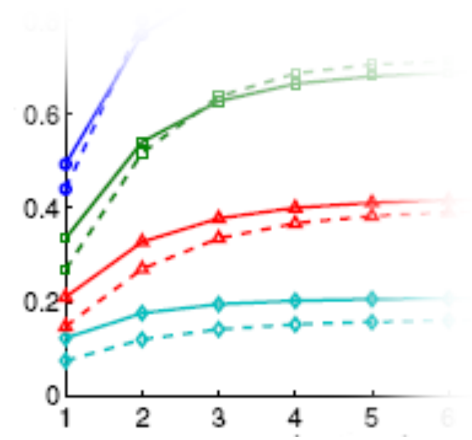
Probabilistic model checking

- Construction and analysis of probabilistic models
  - for example: discrete-time Markov chains (DTMCs)
  - transitions labelled with probabilities
  - from a description in a high-level modelling language
- Correctness properties expressed in probabilistic temporal logic, e.g. PCTL
  - *trigger* $\rightarrow P_{\geq 0.999} [F^{\leq 2}$ *deploy* $]$
  - "the probability of the airbag deploying within 2 time units of being triggered is at least 0.999"



Probabilistic model checking

- Computation of "exact" results (e.g. probabilities)
  - graph algorithms, linear equations, linear programming, numerical fixed points, numerical approximations, ……
- Combines numerical and exhaustive analysis
  - results show system flaws, anomalies



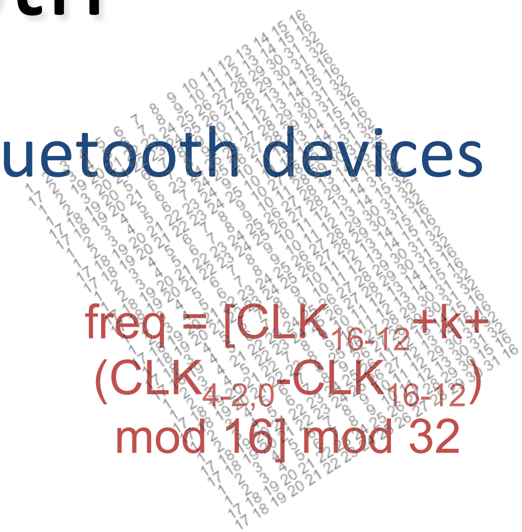
- Flexible and widely applicable
  - many types of models, properties
  - fully automated + tool support
- Scalability and efficiency remains a challenge
  - but many advances in efficient techniques

PRISM

- PRISM: open source probabilistic model checker
  - developed at Birmingham/Oxford University, since 1999
  - wide range of probabilistic models, temporal logics
  - modelling language, GUI, scalable/efficient techniques
- Leading probabilistic verification tool
  - research/teaching in 50+ institutions
  - 34,000 downloads, 250 external PRISM-related papers
- Case studies
  - network protocols, security, biology, robotics, power management, airbag system, cloud computing…
- See: www.prismmodelchecker.org

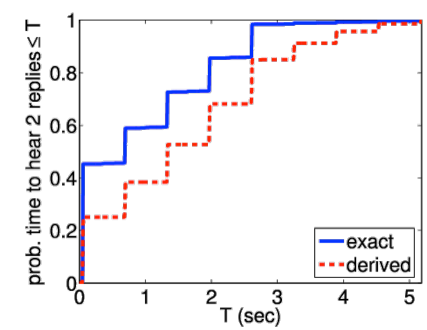
Example: Bluetooth

- Device discovery between a pair of Bluetooth devices
  - performance essential for this phase



$$freq = [CLK_{16-12} + k + (CLK_{4-2,0} - CLK_{16-12}) \bmod 16] \bmod 32$$

- Complex discovery process
  - two asynchronous 28-bit clocks
  - pseudo-random hopping between 32 frequencies
  - random waiting scheme to avoid collisions
  - 17,179,869,184 initial configurations



- Probabilistic model checking (PRISM)
  - "probability discovery time exceeds 6s is always < 0.001"
  - "worst-case expected discovery time is at most 5.17s"

Outline

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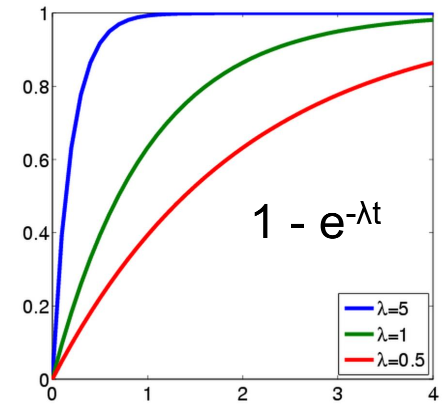
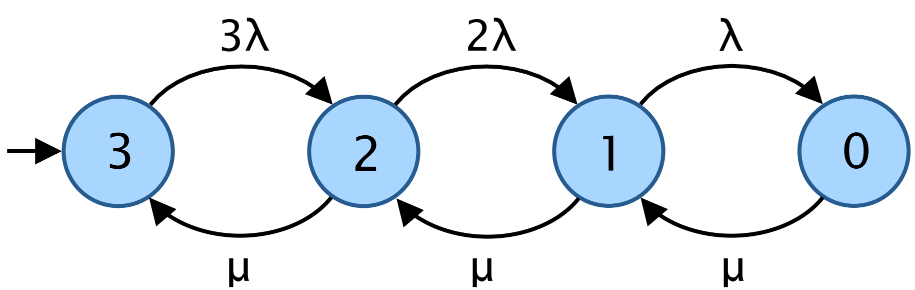
- Adding continuous-time…
  - continuous-time Markov chains
- Adding nondeterminism…
  - Markov decision processes
- Adding game theory…
  - stochastic multi-player games

Adding continuous time…

Adding continuous time…

- Continuous-time Markov chains
  - random (real-valued) transition delays
  - delays are exponentially distributed
  - e.g. failure rates, reaction times, …

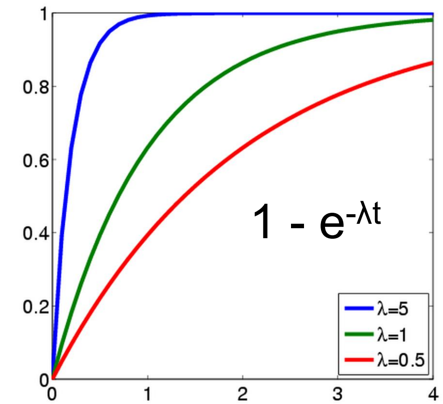


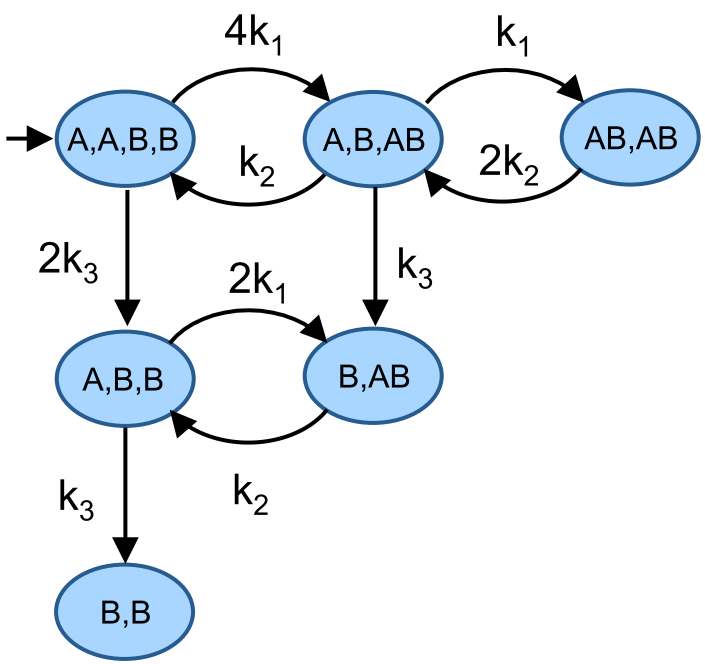


Failures/repairs in a cluster of 3 workstations

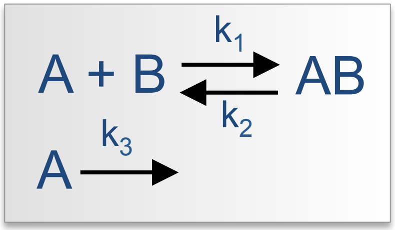
Adding continuous time...

- Continuous-time Markov chains
  - random (real-valued) transition delays
  - delays are exponentially distributed
  - e.g. failure rates, reaction times, ...





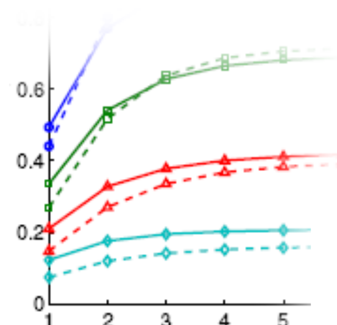
Reactions between proteins A, B & AB

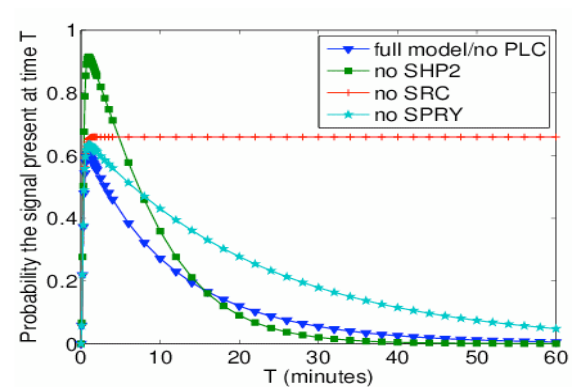


Continuous-time Markov chains

- Properties (temporal logic CSL)
  - $S_{>0.999}$ [*up*] : "long-run probability of availability is >0.999"
  - $P_{=?}$ [*down* $U^{\geq 60}$ *repair*] : "what is the probability that it takes longer than 1 hour to recover from a server failure?"
  - $R^{A}_{=?}$ [$I^{=T}$] : "expected number of molecules of A at time T?"

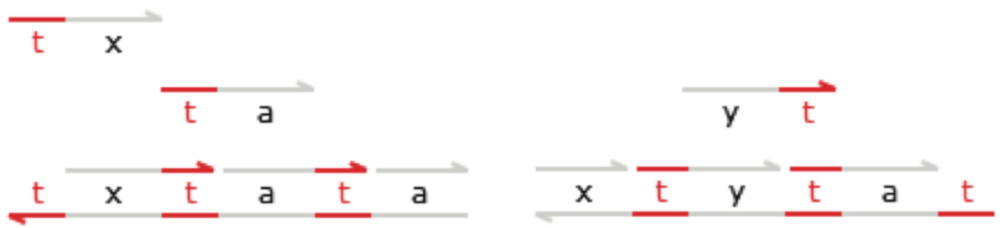
- Applications
  - performance evaluation and reliability analysis
  - systems biology: “in-silico” experiments to validate biologists’ models; later compared to lab results





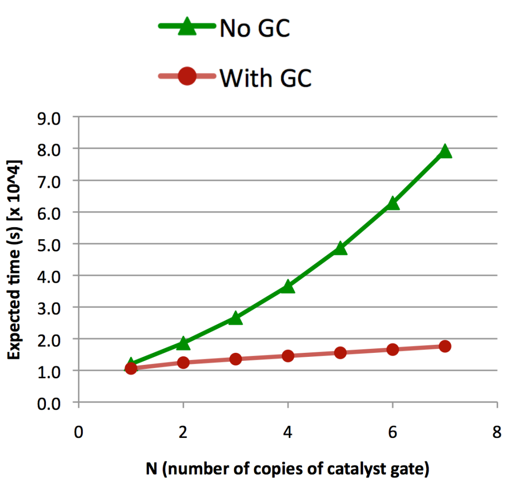
Example: DNA computing

- DNA Strand Displacement language (DSD)
  - for designing DNA circuits [Cardelli, Phillips, et al.]



  - reactions naturally modelled as CTMCs

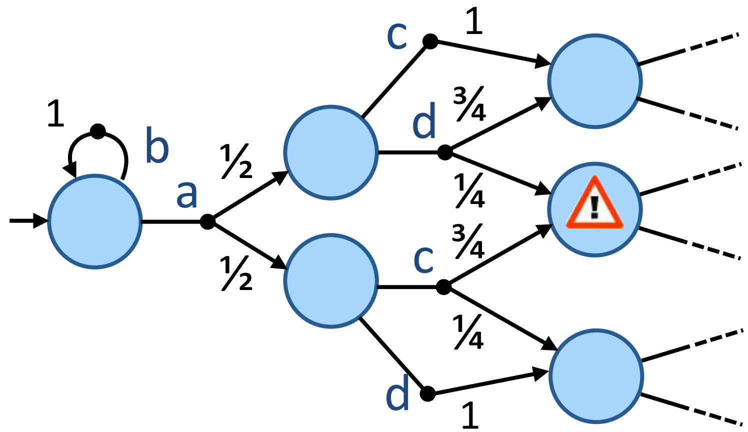
- Analysis of a DNA transducer design
  - correctness: A [G *deadlock* → *all_done*]
    design flaw (due to cross talk) automatically detected
  - performance-based design decisions: with or without garbage collection?



Adding nondeterminism…

Adding nondeterminism...

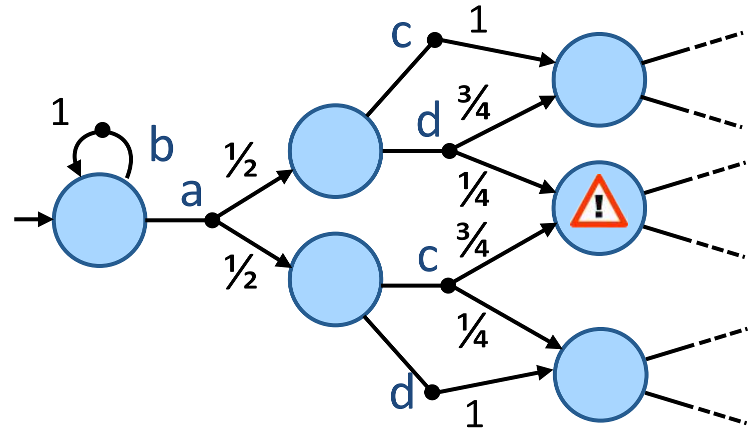
- Markov decision processes (MDPs)
  - generalise DTMCs by adding nondeterminism



- Nondeterminism: unknown behaviour
  - concurrency, abstraction, user input, control
- Strategies (or "policies", "adversaries")
  - resolve nondeterminism based on current history

Markov decision processes

- Two (dual) problems:



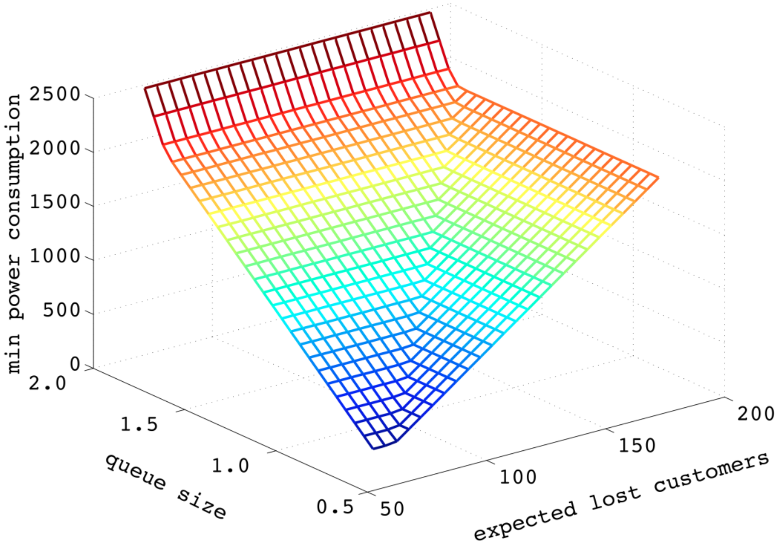
- 1. Verification
  – quantify over all possible strategies (i.e. worst-case)
  – $P_{<0.01}$ [F *err*] : "the probability of error is always < 0.01"
  – applications: randomised communication protocols, randomised distributed algorithms, security, …

- 2. Strategy synthesis
  – $P_{<0.01}$ [F *err*] : "does there exist a strategy for which the probability of an error occurring is < 0.01?"
  – applications: robotics, power management, security, …

Example: Power management

- Dynamic power management controllers
  - for an IBM TravelStar VP disk drive
  - switch between power modes: active/idle/idlelp/stby/sleep
  - PRISM model of power manager, disk request queue, etc.

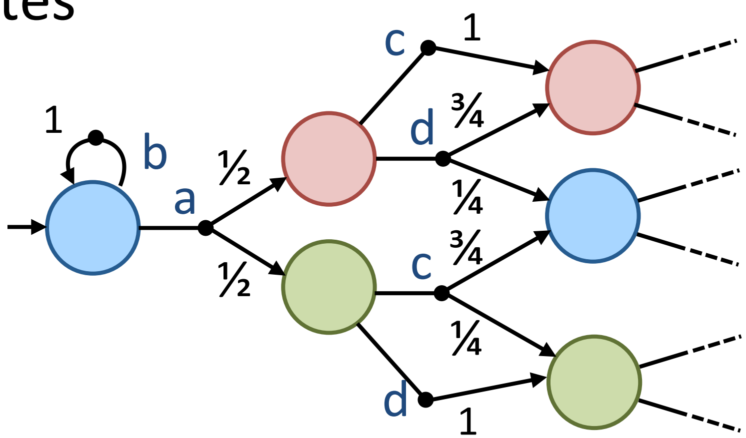
- Build controllers that
  - minimise energy consumption, subject to constraints on e.g.
  - (i) probability that a request waits more than K steps
  - (ii) expected number of lost disk requests



Adding game theory…

Adding game theory…

- Stochastic multi-player games
  - states controlled by players
  - players choose actions in states
  - strategies for each player



- Key ideas
  - models competitive and/or collaborative behaviour
  - automated methods essential to reason about complex player strategies, and interaction with probabilities

Stochastic multi-player games

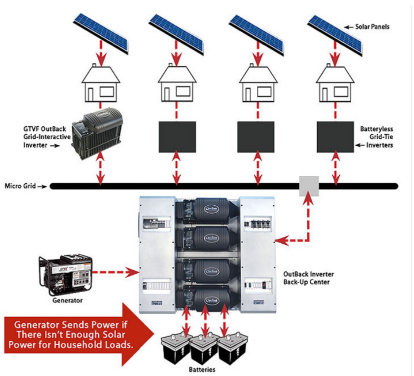
- Property specifications (temporal logic rPATL)
  - $\langle\langle\{1,2\}\rangle\rangle\ P_{\geq 0.95}\ [\ F^{\leq 45}\ \mathit{done}\ ]$: "can nodes 1 and 2 collaborate so that the probability of the protocol terminating within 45 seconds is at least 0.95, whatever nodes 3 and 4 do?"

- Model checking
  - zero sum properties: analysis reduces to 2-player game
  - PRISM-games: www.prismmodelchecker.org/games

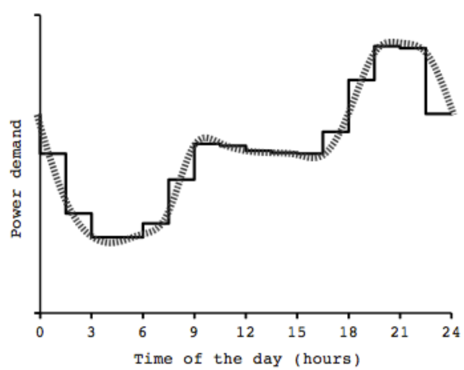
- Applications
  - controller synthesis (controller vs. environment), security (system vs. attacker), distributed algorithms, …

Example: Energy management

- Energy management protocol for Microgrid
  - Microgrid: local energy management
  - randomised demand management protocol
  - random back-off when demand is high



- Original analysis [Hildmann/Saffre'11]
  - protocol increases "value" for clients
  - simulation-based, clients are honest

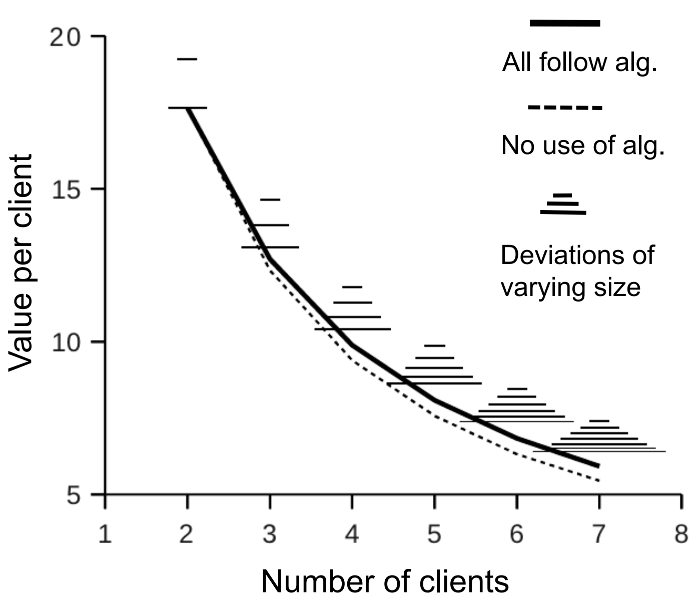


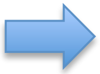
- Stochastic multi-player game model
  - clients can cheat (and cooperate)

Example: Energy management

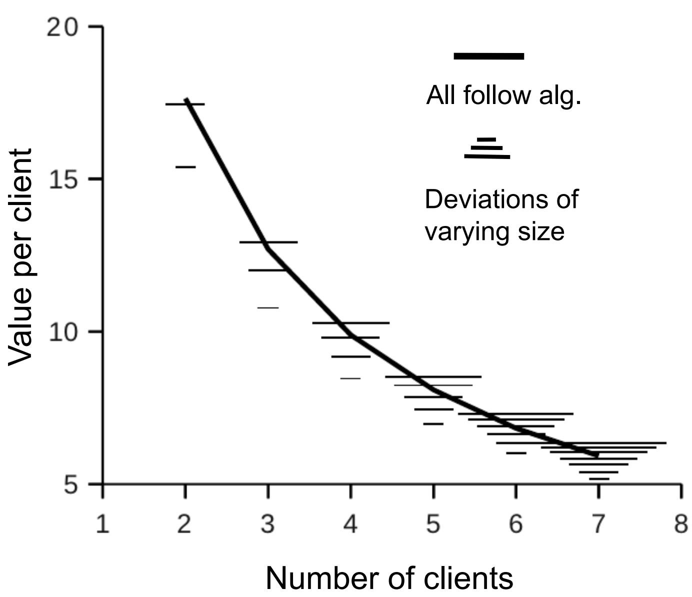
- Exposes protocol weakness
  - incentive for clients to act selfishly
- We propose a simple fix (and verify it)
  - clients can be punished

Value per client





Value per client, with fix



Conclusions

- Quantitative verification
  - probabilistic model checking & PRISM
  - formal methods to build/analyse probabilistic models
  - temporal logics for correctness, reliability, performance, ...
  - exact results, combines numerical + exhaustive analysis
  - flexible approach, wide range of applications
- Key challenges
  - scalability + efficiency: state space explosion
  - richer models: continuous space, hybrid systems, ...
  - user friendly languages for model/property specification