Quantitative Verification: Correctness, Reliability and Beyond

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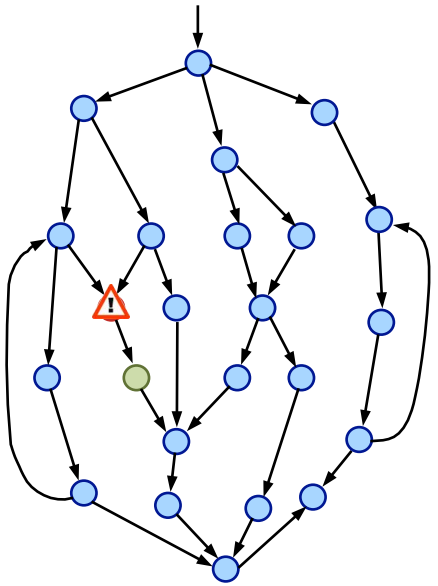
December 2012

Verification

- Checking the correctness of (computerised) systems using rigorous, mathematically-sound techniques
  - in essence: proving that a piece of software, or hardware, or a protocol behaves correctly

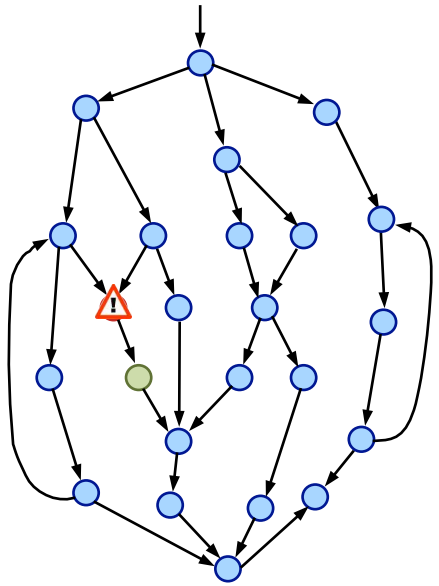
- Automated verification: model checking
  - correctness properties expressed in temporal logic
  - exhaustive construction/analysis of finite-state model



$\forall$ [$\Box$(*warn* $\rightarrow$ $\bigcirc$ *trigger*)]

Model checking

- Successful in practice
  - e.g. Windows device drivers, circuit designs, …
- Example properties
  - "acquire/release of spinlock is always done in strict alternation"
  - "no array is accessed outside its bounds"
- Why it works
  - temporal logic: expressive, tractable
  - fully automated, tools available
  - not just verification, but falsification of properties, i.e. bug hunting



$\forall$ [$\Box$(*warn* $\rightarrow$ $\bigcirc$ *trigger*)]

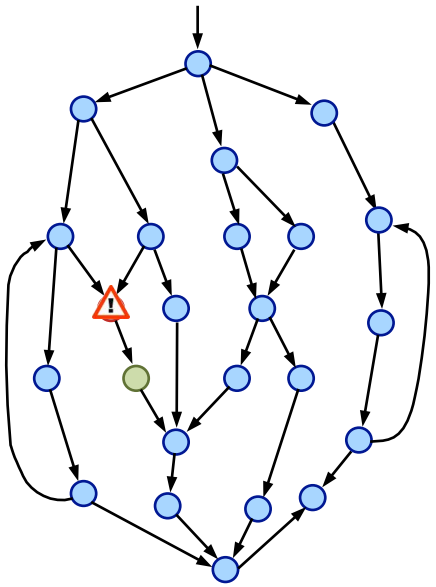
Quantitative verification

- Adds quantitative aspects (to models and properties)
  - probability, time, costs, rewards, …
- Probability
  - physical components can fail
  - communication media are unreliable
  - algorithms/protocols use randomisation
- Time
  - delays, time-outs, failure rates, …
- Costs & rewards
  - power consumption, resource usage, …
  - profit, incentive schemes, …

Probabilistic model checking

- Construction and analysis of probabilistic models
  - Markov chains, Markov decision processes, …
- Correctness properties in probabilistic temporal logic
  - $P_{>0.999}$ [$\Box(\mathit{trigger} \rightarrow \Diamond^{\leq 20}\, \mathit{deploy})$]
  - "the probability of an airbag always deploying within 20ms of being triggered is at least 0.999"
  - correctness, reliability, performance, …
- Model checking algorithms (and tools)
  - graph algorithms, linear equations, linear programming, numerical fixed points, numerical approximations, …

Probabilistic models



+ probabilities

Markov chain

e.g. communic-ation procotol

+ exponential

time delays

continuous-time Markov chain

e.g. systems biology

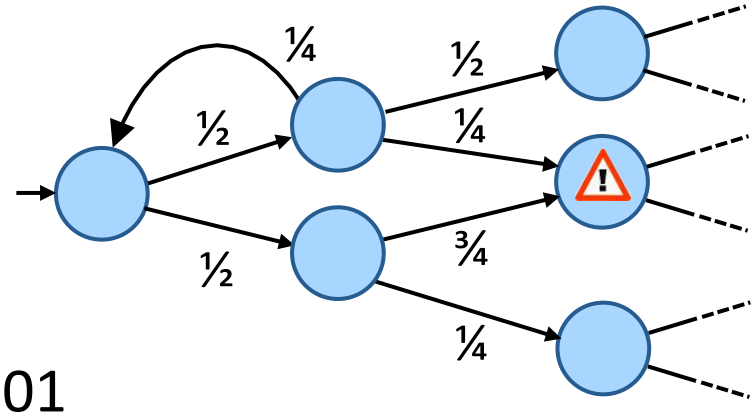
+ game theory

stochastic game

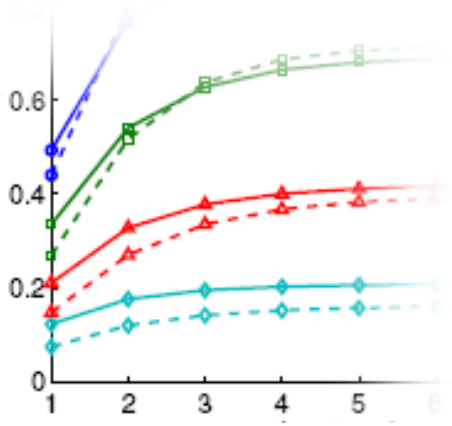
e.g. energy management

1) Adding: Probabilities

- Model: Markov chain
  - add probabilities to transitions



- Properties
  - probability of airbag failure < 0.001
  - numerical queries: what is the probability of failure?
- Key ideas:
  - exact numerical results
  - combines numerical + exhaustive analysis
  - results show system flaws, anomalies

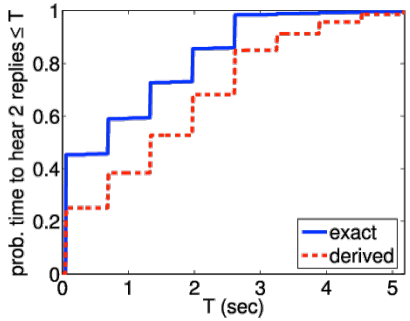


- Applications
  - network protocols, security, biology, robotics & planning, power management, nanotechnology…

Example: Bluetooth

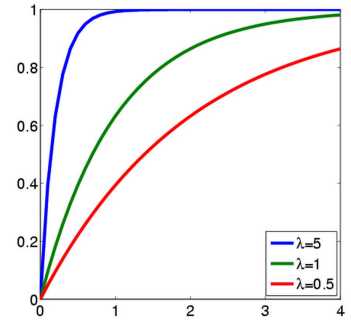
- Device discovery between a pair of Bluetooth devices
  - performance essential for this phase
- Complex discovery process
  - two asynchronous 28-bit clocks
  - pseudo-random hopping between 32 frequencies
  - random waiting scheme to avoid collisions
  - 17,179,869,184 initial configurations
- Probabilistic model checking
  - “worst-case expected discovery time is at most 5.17s”
  - “probability discovery time exceeds 6s is always < 0.001”

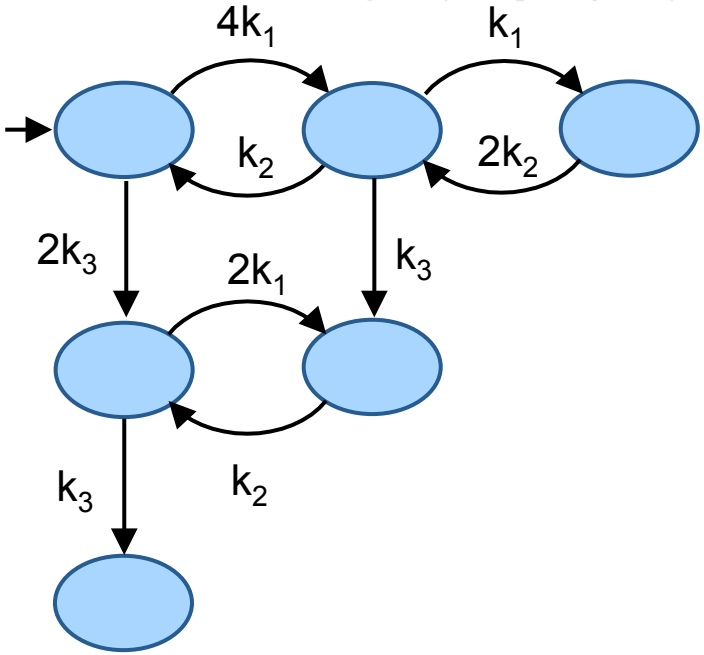
$$\text{freq} = [CLK_{16\text{-}12} + k + (CLK_{4\text{-}2,0} - CLK_{16\text{-}12}) \bmod 16] \bmod 32$$



2) Adding: Exponential delays

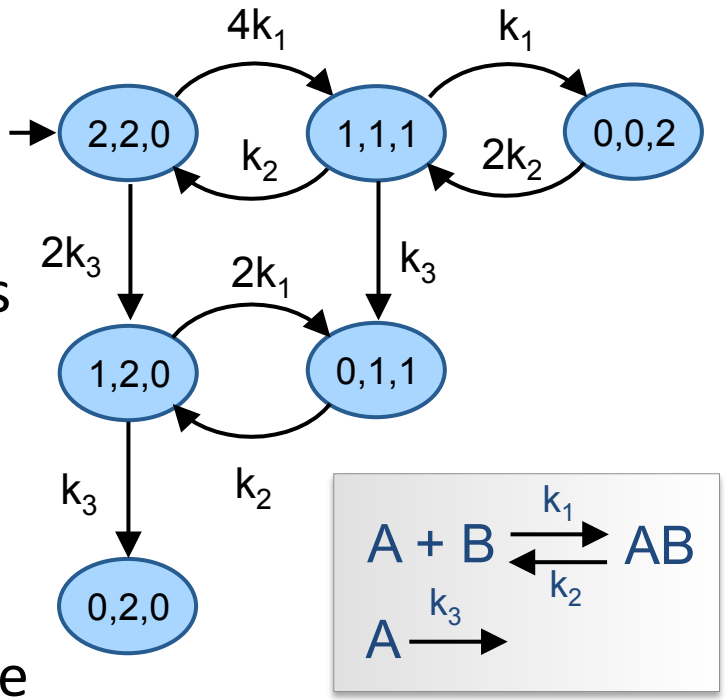
- Continuous-time Markov chains
  - random delays on transitions between states
  - delays are exponentially distributed
  - e.g. failure rates, reaction times, …
- Applications
  - network performance models
  - biological reactions
- Properties
  - probability of disk-failure within 1 month?
  - expected number of molecules of X at time instant T?

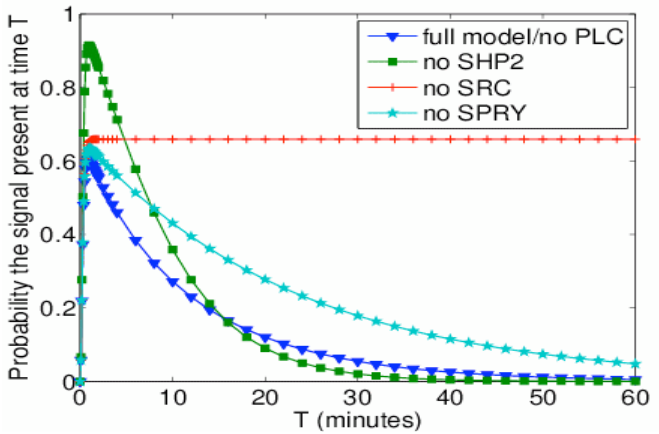




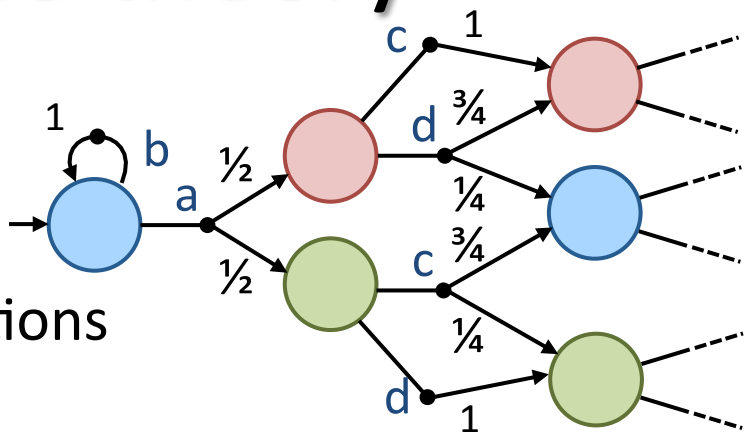
Example: Systems biology

- Markov model of reactions
  - states represent molecule counts
  - transitions correspond to reactions
- Key ideas
  - "in-silico" experiments
  - aim: validate biologists' models
  - probabilistic model checking can be cheaper than simulation
  - small models yield useful results
- Case study: FGF pathway
  - model developed with biologists
  - validated against lab experiments





3) Adding: Game theory

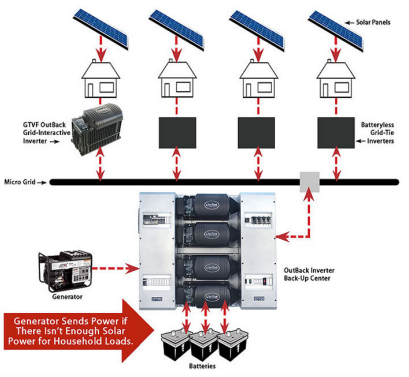


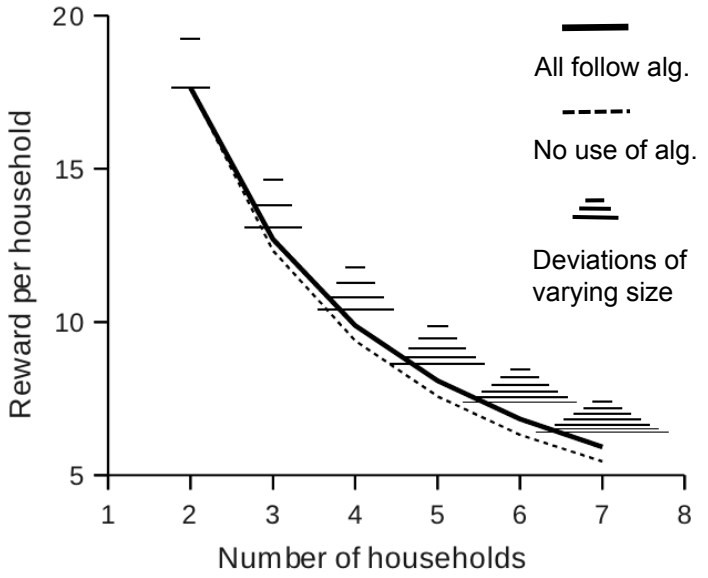
- Multi-player stochastic games
  - states controlled by players
  - players choose (probabilistic) actions
- Key ideas
  - automated methods essential to reason about complex player strategies, and interaction with probabilities
- Property specifications
  - does player 1 have a strategy to ensure that the probability of is <0.01, regardless of the strategies if players 2 and 3?
- Applications
  - controller synthesis (controller vs. environment), security (system vs. attacker), distributed algorithms, …

Example: Energy management

- Energy management protocol for Microgrid
  - Microgrid: local energy management
  - randomised demand management protocol
  - probability: randomisation, demand model, …
- Existing analysis
  - simulation-based
  - assumes all clients are unselfish
- Our analysis
  - stochastic multi-player game
  - clients can cheat (and cooperate)
  - exposes protocol weakness
  - propose/verify simple fix

[Hildmann/ Saffre'11]





Conclusions

- Quantitative verification
  - formal methods to build/analyse probabilistic models
  - temporal logics for correctness, reliability, performance, …
  - exact results, combines numerical + exhaustive analysis
  - wide range of applications

- Challenges
  - scalability + efficiency
  - wider property classes, e.g. partial information for games
  - richer models: timed games, hybrid automata, …