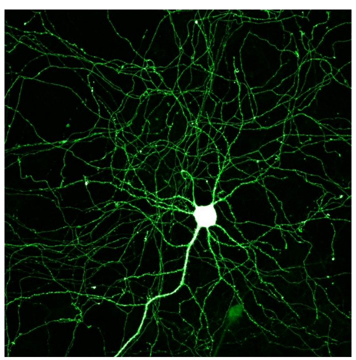
Computational resources to generate correlations

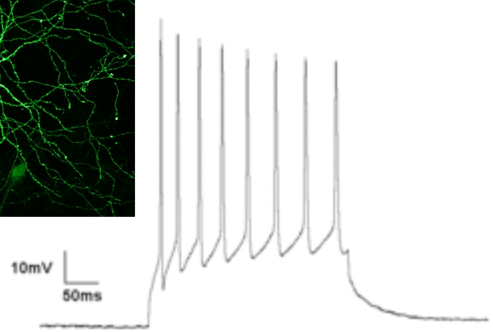
Karoline Wiesner

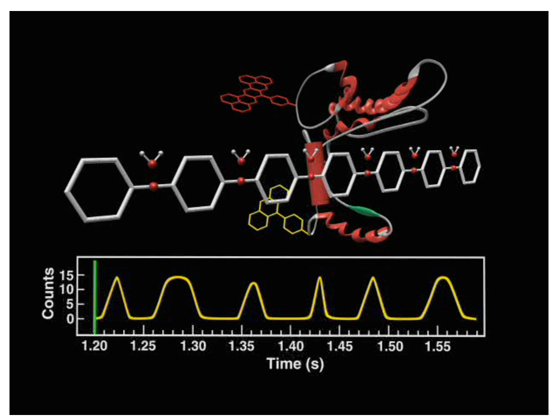
School of Mathematics
University of Bristol

Information processing in living systems

- Information takes the form of statistics and dynamics of patterns over the system's components / evolution. E.g. neural spike sequence distribution, immune system's distribution of lymphocytes, ant colony's spatial distribution, cell's chemical concentrations during metabolism.





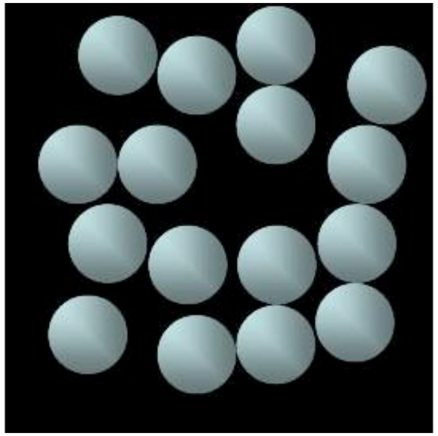


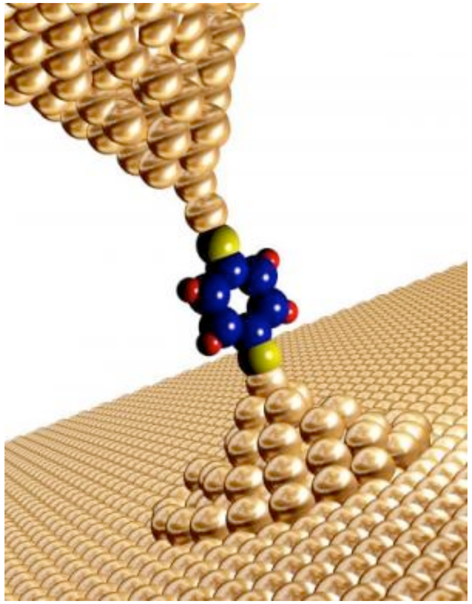
Randomness is free

- The resources needed for stochastic computation need to be counted in a meaningful way. If correlations are the end then what exactly are the means?

- Correlations are expensive.

Correlations Matter



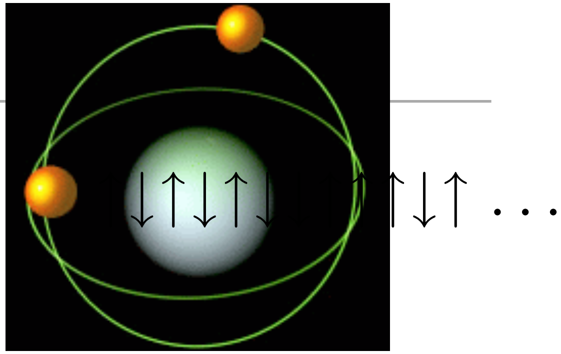


left. http://www.wayoftao.com/pages/tao-and-the-masters/tao-and-science.php
middle. US National Library of Medicine (immunoglobulin G (IgG))
right. http://www.blogsforcompanies.com/TTimages/organic_molecule_creating_current.jpg

GENERATING CORRELATIONS REQUIRES A COMPUTATION

Natural systems exhibit correlations. They are **structured**.
Both in space and in time.

Generation of correlation / structure requires an **intrinsic computation**. Identify the necessary (quantum) computational architecture.



Electronic excitations

How much memory is stored: Quantum excess entropy (Crutchfield, KW, Physics Letters A, 2008).
How is it stored: Quantum computational architecture of quantum finite-state machines (KW, Crutchfield, Physica D, 2008).
Generalised quantum finite-state machines (Alex Monras, KW, Almut Beige (in preparation)).

Generating correlations using stochastic finite-state machines

- Take the perspective:

$$\text{observed behaviour} \leftrightarrow \text{process language} \leftrightarrow \text{output of automaton}$$

- Computing word probabilities

$$Pr(s^L) = \pi^s \, T(s^L) \, \eta$$

  - π^s is the asymptotic state distribution. η is a column vector of 1s.

Correlations as Language

- Take the output as stochastic language

$$\mathcal{L} = \{w \,|\, w = s_0 s_1 s_2 \dots s_{L-1} \in \mathcal{A}\}$$

- A stochastic language is a formal language with a word distribution that is normalised at each length L :

$$\sum_{s^L \in \mathcal{L}} Pr(s^L) = 1 \;,\; L = 1, 2, 3, \dots$$

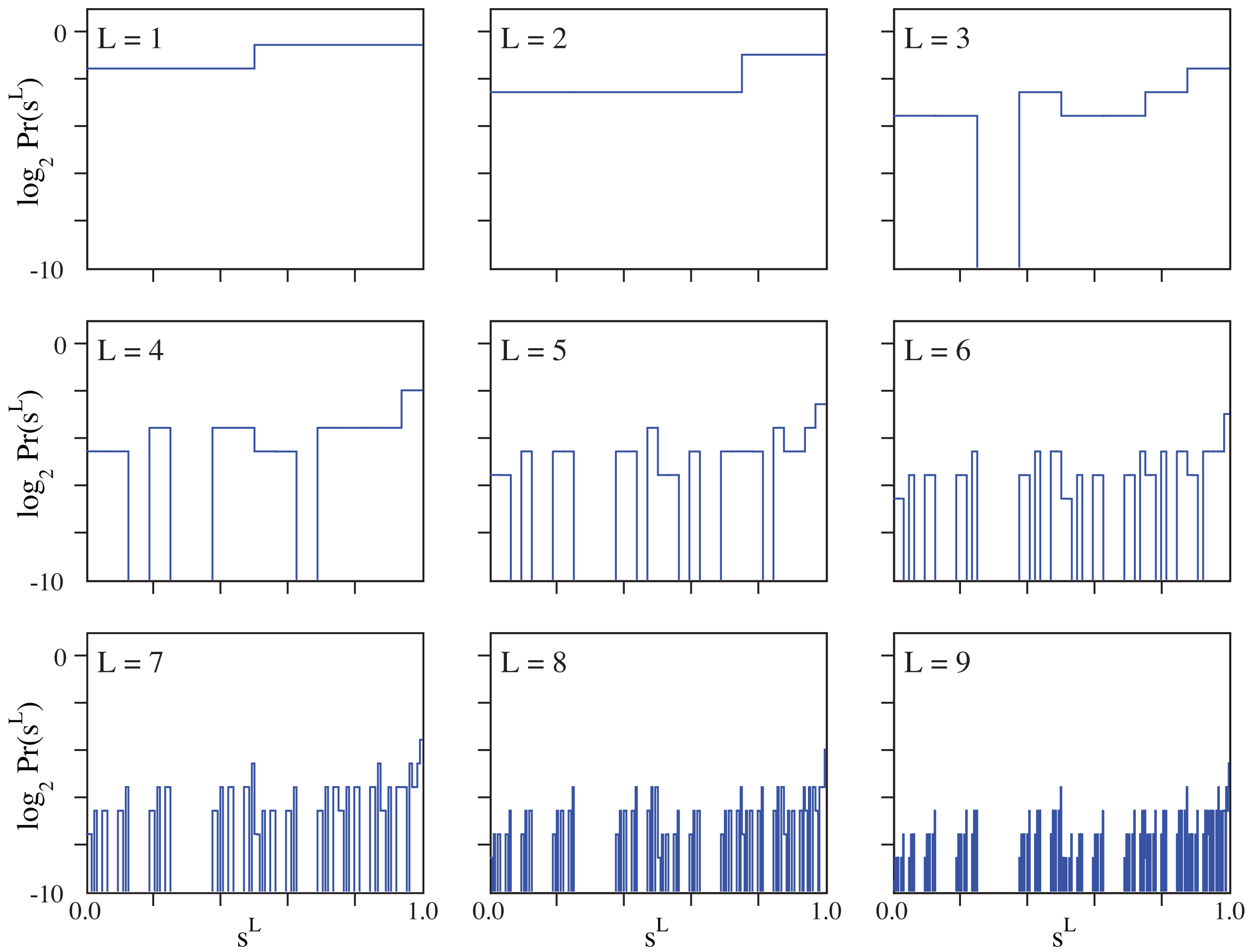
$$Pr(s^L) \geq Pr(s^L s)$$

- A process language is a stochastic language that is sub-word closed.

Example: Even process

- Word distribution of Even Process

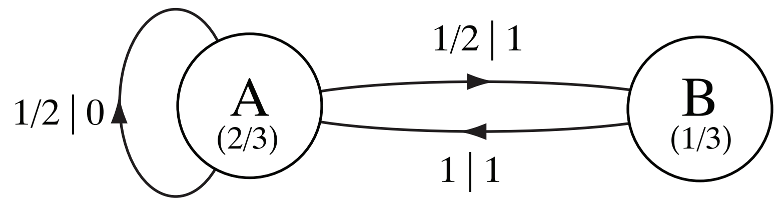
First forbidden word: 010



$$h_\mu = 2/3$$
$$E = 0.902$$

Even process ε - machine

- ...110001100011110110001111 0...



- This process has an infinite memory, it is a strictly *sofic* system.

Complexity of the ε - machine

- The e-machine is provably minimal and optimal for a given probability distribution [Crutchfield, Young 1989].

- The size of the e-machine (Shannon entropy over state distribution) is the complexity of the process.

$$C_\mu = -\sum_{\{S\}} Pr(S_i) log_2 Pr(S_i)$$

Quantum Process

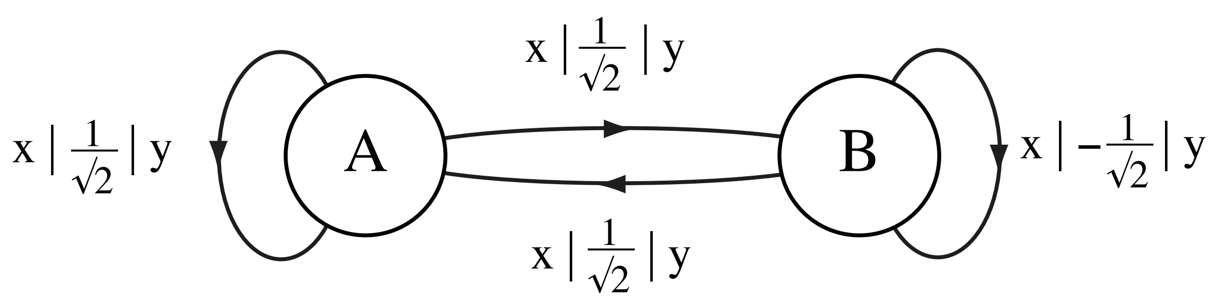
- System $\langle\psi|$, unitary dynamics U, general measurement operators P.
- Main difference to classical process is that the measurement has to be made explicit!
  - Input: measurement operation
  - Output: measurement outcome
- Observations (spin component, electronic state, ...) yield symbol sequence.

Quantum finite-state machine

- Computation-theoretic representation of a quantum process:

Quantum finite-state transducer.

$$\{\{Q\}, \mathcal{H}, y \in \mathcal{Y}, \{T(y|x)\}\}$$



$$\begin{array}{c|c|c} \textit{input symbol} & \textit{transition amplitude} & \textit{output symbol} \\ x & [T(y|x)]_{ij} & y \end{array}$$

"Computation in finitary stochastic and quantum processes", KW, JP Crutchfield, Physica D, 2008.

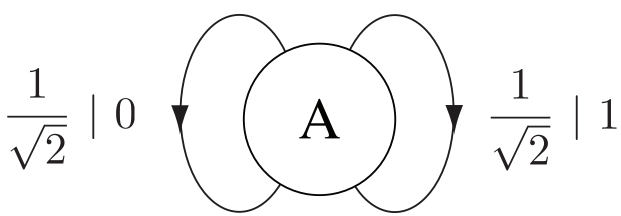
Quantum Processes

- Definition of a quantum finite-state generator [KW, Crutchfield, 2007].
- Can **any classical process** be generated by a quantum finite-state machine?
    - **No**, if unitary evolution + projection measurement [Crutchfield, KW, 2008]
    - **Yes**, if generalised measurement [Monras, Beige, KW, in prep.]
- Identify a hierarchy of language classes generated by quantum machines [KW, Crutchfield, 2007].

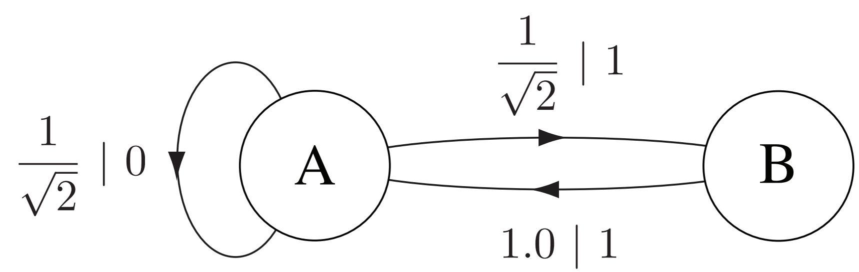
Quantum finite-state generator

- Generator: only one input
    - States, transitions with amplitudes, output
- Examples:

$$X_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} \end{pmatrix} \qquad X_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \end{pmatrix}$$



$$X_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \qquad X_1 = \begin{pmatrix} 0.0 & 1.0 \\ \frac{1}{\sqrt{2}} & 0.0 \end{pmatrix}$$



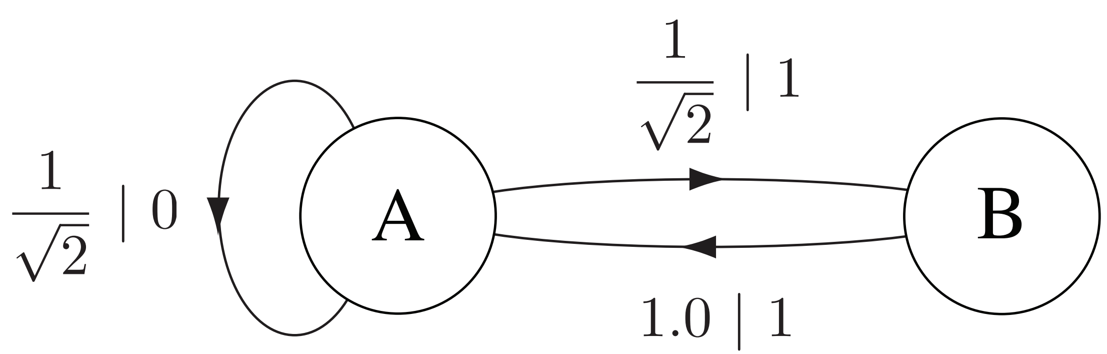
Quantum processes

Quantum Even Process

...11000110001111011000111110...

This process has an infinite memory, it is a strictly *sofic* system.

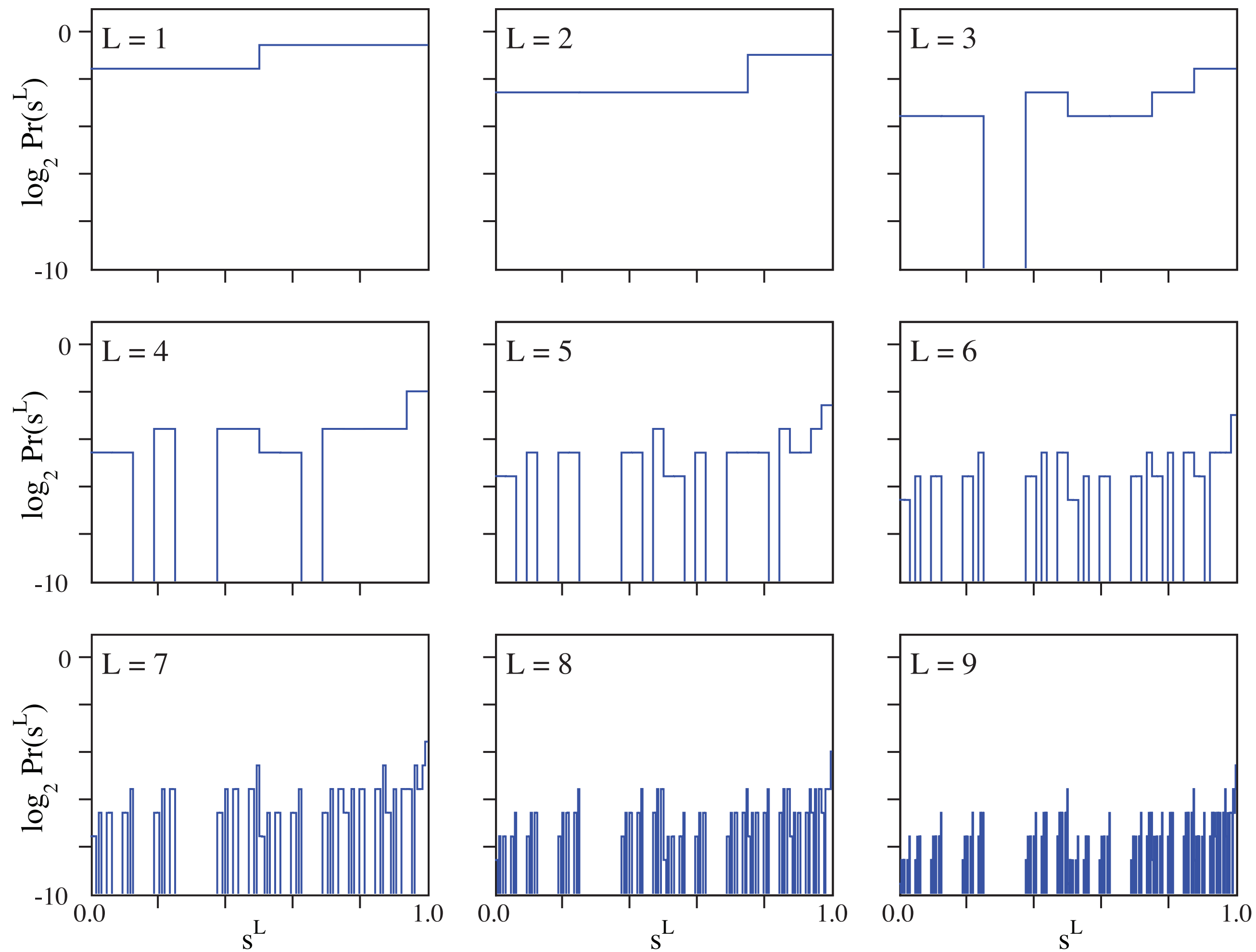
$$X_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \qquad X_1 = \begin{pmatrix} 0.0 & 1.0 \\ \frac{1}{\sqrt{2}} & 0.0 \end{pmatrix}$$



Example: Quantum Even process

Word distribution of Quantum Even Process

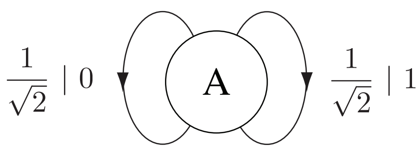
First forbidden word: 010



$$h_\mu = 2/3$$
$$E = 0.902$$

Entropy rate and excess entropy for some deterministic quantum processes:

$$X_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} \end{pmatrix} \quad X_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \end{pmatrix}$$



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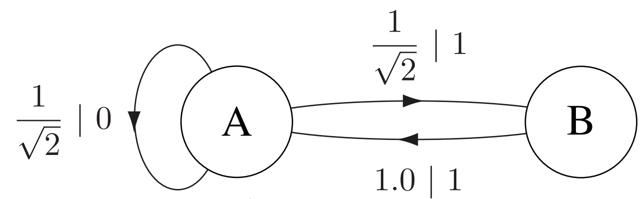
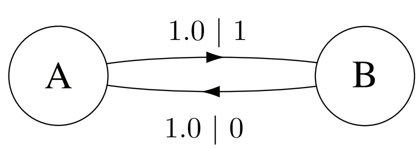
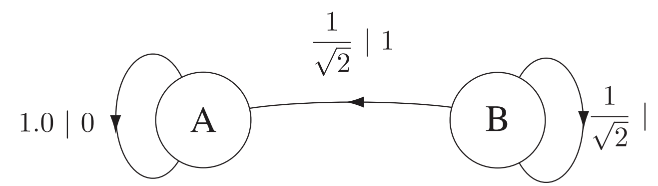


Process	Fair Coin	Period-2	Even	Amplitude Damping
h_μ [*bits/measurement*]	1.0	0.0	0.666	0.0
E [*bits*]	0.0	1.0	0.902	2.0

$$X_0 = \begin{pmatrix} 0.0 & 1.0 \\ 0.0 & 0.0 \end{pmatrix} \quad X_1 = \begin{pmatrix} 0.0 & 0.0 \\ 1.0 & 0.0 \end{pmatrix}$$



$$X_0 = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & \frac{1}{\sqrt{2}} \end{pmatrix} \quad X_1 = \begin{pmatrix} 0.0 & \frac{1}{\sqrt{2}} \\ 0.0 & 0.0 \end{pmatrix}$$



Information Theoretic Measures

- Use Shannon entropy to obtain entropy rate as measure of information production

$$h_\mu = \lim_{L\to\infty} [H(L) - H(L-1)]$$

  - Units: bits/measurement
  - Measure of entropy growth / information production.
  - Measure of intrinsic unpredictability.

Information Theoretic Measures (cont.)

- Use entropy rate to obtain excess entropy

$$E = \lim_{L \to \infty} [H(L) - h_\mu L]$$

  - Units: bits
  - *Apparent* randomness that is eventually explained by longer data sequences.
  - How much of its past does the process store.
  - Mutual information between semi-infinite past and semi-infinite future:

$$E = \lim_{L \to \infty} I(\overleftarrow{S}^L ; \overrightarrow{S}^L)$$

Summary

Correlations matter (spin systems, non-linear dynamical systems, neural signals). How are correlations computed?

Correlations are statistical hence we need stochastic computational models. The simplest models are stochastic finite-state machines. They can be inferred in provably minimal and optimal form. In principle they can be used to compute correlations of measured quantum systems. This automatically classifies (measured) quantum systems in terms of languages. But it seems an obvious step to define quantum finite-state machines.

Quantum automata with projective measurement => only a subset of the stochastic regular languages can be generated. Quantum automata with generalised measurement => all stochastic regular languages can be described.

Acknowledgements & References

James Crutchfield, John Mahoney (UC Davis), Almut Beige, Alex Monras (U Leeds)

KW and J.P. Crutchfield, *Computation in finitary stochastic and quantum processes*, quant-ph/0608206, Physica D, 2008.

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Alex Monras, KW, Almut Beige, *Generalised quantum finite-state machines,* (in preparation)